ARITHMETIC BY GRADES

FOR INDUCTIVE TEACHING, DRILLING AND TESTING

BOOK NUMBER EIGHT

Miscellaneous Questions involving the making of definitions, rules and formulas, Algebraic Exercises, Involution and Evolution, Exercises in Geometry and Mensuration Book-keeping, Miscellaneous Reviews

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NOTE TO TEACHERS.

THE attention of teachers is called to the following features of this series of books — features which should be kept in mind as the various subjects are presented.

1. The separation of teachers' and pupils' books, whereby pupils may be taught properly and may not be given too great assistance. Suggestions as to methods of teaching and drilling, as well as the illustrative processes, explanations, rules, and definitions which belong to the teacher to develop analytically are put into the Teachers' Manual, while in the pupils' books are presented only such exercises as are needed for practice.

2. The careful gradation of problems, by which pupils acquire inductively a knowledge of arithmetical relations and principles, and skill in arithmetical processes. This is in recognition of the well-known pedagogical principles of proceeding from the known to the unknown, and from the simple to the complex. It is advised that this plan be kept constantly in mind by the teacher, and that whenever a process is not understood or is not readily performed, the pupils should be taken back to processes which are well known and which can be performed readily, and then should be led forward by easy steps until the desired end is reached.

3. Frequent reviews, and such an arrangement of exercises as will enable pupils to have needed practice in the applications of each principle, first by itself, and afterwards in connection with other principles which have been learned.

4. The large amount of oral work, or work which may be done without the aid of figures. Three objects of Mental Arithmetic are sought in these exercises: (*a*) Illustration of principles and a preparation for written work, (*b*) Development of the logical powers, (*c*) Cultivation of ability to work with large numbers by short processes.

5. The great number and variety of problems. The aim has been to give the *largest number* of problems that will be needed for teaching and for drilling in all grades. For this reason, and because the forms of expression are varied, being taken from many sources, there will be no necessity of giving supplementary drill lessons on the blackboard. Blackboard lessons are objectionable not only on account of a waste of the teachers' time and strength, but also on account of the injury done to pupils' eyes in much reading and copying from the blackboard.

6. Practicalness of work in respect to the character of the problems, and the solution of them. Care has been taken to give problems which are most likely to be met in every-day life, and to give them in a practical form. Many of the miscellaneous review problems were made by mechanics, clerks, accountants, etc., with a view of presenting conditions most likely to occur.

7. The introduction of statistics and facts of physics, astronomy, history, geography, etc., thus enabling pupils to gain incidentally much useful information.

8. The use of drill tables and other devices to save the time of teachers.

In addition to the above features, some of which are distinctively new so far as American text-books are concerned, there is the separation of pupils' exercises for practice into small books somewhat on the lines of gradation in City graded schools. By this arrangement there are gained greater convenience of handling and economy of wear than in the use of a large book which is intended to be used for several years by the same pupil.

Much of the arithmetical part of this book is supposed to be a review of what has been taught. There is little, therefore, of what may be called development work. If such work is found necessary, as it may be for the solution of the most difficult problems, it should be supplied by the teacher.

Rules and definitions which have not been called for to a great extent in previous books should be required, not merely as a test of the pupil's knowledge, but as a good means of mental discipline. Let the rules and definitions be made by the pupils themselves, and be corrected by them as they see that other forms of expression are clearer or more accurate than those which they have made.

In the exercises in Algebra an effort has been made to introduce such operations in simple equations as will lead to a familiarity with algebraic expressions, to a better understanding of principles, and to an ability to perform certain problems which are found too difficult to be solved by arithmetic.

While the inventional and constructive side of mensuration and geometry is continued, somewhat of demonstrative work may be introduced, and definitions which have not before been used to a great extent should now be required frequently. Particular attention should be given to the practical applications of both Geometry and Algebra.

The exercises in Book-keeping are believed to be practical and sufficient for any business which does not require an elaborate system of double entry.

For methods of teaching the various subjects and for answers to problems, see Teacher's Manual, which is designed to accompany all books of the series.

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SECTION I.

Miscellaneous Questions and Exercises.

1. One chair, one crayon, one pair of gloves, one-half of an apple, are examples of a unit. Give other examples. Define unit; integral unit; fractional unit.

2. Give examples of a number. Define number; integral number; fractional number.

3. What knowledge is gained by the study of arithmetic? Give examples of expression of numbers; operations of numbers; relations of numbers. Define arithmetic.

4. Upon what principle does the decimal system of numbering depend? Show by examples. Illustrate other systems.

5. Find the origin of the names of numbers one, two, etc., to one hundred.

6. Illustrate with objects the four fundamental processes — addition, subtraction, multiplication, and division, and define each.

7. Give examples and define sum; minuend: subtrahend; remainder; multiplicand; multiplier; product; factor; dividend; divisor; quotient.

8. Illustrate and define odd number; even number; prime number; composite number; prime factor; multiple; common multiple; least common multiple; common divisor; greatest common divisor.

9. Illustrate and define denominator; numerator.

10. Illustrate the process of reducing fractions to their lowest terms, and give the rule.

11. Give the rule for changing fractions to equivalent fractions having the least common denominator.

12. Illustrate by objects or drawings the process of adding fractions. Give the rule for adding fractions.

1. Illustrate the process of subtracting fractions, and give the rule.

2. Illustrate and give the rule for multiplying a fraction by an integer; for finding the fractional part of a number.

3. Illustrate and give the rule for dividing an integer by a fraction; for dividing a fraction by a fraction.

4. Write a number involving billions and billionths. Numerate it. Illustrate the effect of removing the decimal point to the right; to the left. Illustrate the effect of annexing ciphers to a decimal.

5. Give the rule for reducing a common fraction to a decimal; for reducing a decimal to a common fraction.

6. Give the rule for multiplying decimals; for dividing decimals.

7. Write with abbreviations the tables of long measure — common, surveyors', circular, and metric. What is the standard unit in each? How many feet in a mile? How many yards?

8. Explain the following terms: geographical mile; statute mile; knot; league; furlong; fathom; cable length; span; hand.

9. How is the degree found? Its length is how determined? What is the length of a degree of latitude? What is the length of a degree of longitude on the equator? on the 40th parallel? on the 60th? on the 30th?

10. Describe the Gunter's chain; the surveyor's steel ribbon.

11. Show by examples how measures of surface are found. Write with abbreviations the tables of square measure — common, surveyors', and metric. How many square feet in an acre? how many square yards?

12. Show by examples how measures of volume are found. Write with abbreviations the tables of cubic measure, common and metric. What is meant by a cord foot? by a perch of masonry?

13. Write with abbreviations the tables of weight — avoirdupois, apothecaries', and troy; also by metric system. For what is each used? What is the difference in weight between the pound troy *and* the pound avoirdupois? between the ounce troy and the ounce *avoirdupois?* What is the long ton, and when is it used?

1. What is the standard unit of dry measure? of liquid measure? For what is dry measure used? liquid measure? Write with abbreviations the tables of each, in the common and metric systems. How many cubic inches in a bushel? in a gallon? Explain the following terms: chaldron; cental; barrel; tierce; hogshead; pipe or butt; tun; stone. What is the weight of a barrel of flour? of beef or pork? of a bushel of wheat, corn, oats, etc.?

2. What is the standard of measure by the metric system? How obtained?

3. Show the connection of all weights and measures by the metric system, and the meaning of all prefixes.

4. Show by illustrations the advantage to be gained by the use of the metric system of weights and measures.

5. Give the equivalents in United States measures of: the meter; kilometer; square kilometer; ar; hektar; ster; liter; hektoliter; gram; kilogram; metric ton.

6. Write with abbreviations the table of time. What is meant by a solar year? Exact length of a solar year? Why is every fourth year one day longer? How can you tell what years are leap years? What is meant by a calendar year? How many days in each month of the year? What is meant by a lunar month? Show by examples how you estimate the difference of time in years, months, and days between two dates. How in exact number of days? Show by examples the relations between longitude and time.

7. Name and give the value of each of the principal coins of England; of Germany; of France; of Spain; of Russia; of Mexico. Give equivalents in United States money of the sovereign; shilling; franc; mark; Mexican dollar; rouble.

8. Explain each of the following terms: dozen; gross; great gross; score; quire; ream; bundle; bale; folio; quarto; octavo; duodecimo.

9. Give original problems involving the process of the changing of compound numbers to simple numbers; the changing of simple numbers to compound numbers. Give the rule in each case.

1. In the statement, 8% of \$200 is \$16, what is the base? rate per cent? percentage? amount? remainder? Define each term.

2. Give examples in which the base and rate per cent are given to find the percentage. Give the rule.

3. Give examples in which the base and percentage are given to find the rate per cent. Give the rule.

4. Give examples in which the percentage and rate per cent are given to find the base. Give the rule.

5. Define each of the following terms; commission; consignment; consignor; consignee; insurance; policy; premium; stock; dividend; certificate of stock; bond; coupon; tax; poll tax; real estate; personal property; principal; interest; bank; avails or proceeds; bank discount; promissory note; face of note; maker or drawer of note; payee; holder of note; compound interest.

6. In all applications of percentage show what classes of problems require the percentage to be found, the base and rate per cent being given.

7. Show what classes of problems require the rate per cent to be found, the base and percentage being given.

8. Show what classes of problems require the base to be found, the percentage and rate per cent being given.

9. Show what classes of problems require the base to be found, the amount or remainder and rate per cent being given.

10. Give and solve an example in assessment of taxes, and explain each step of the process. Give a rule for finding the rate of property tax; for finding each individual's tax.

11. Give two ways of computing simple interest.

12. Give rules for finding the balance due when partial payments have been made.

13. Give a problem in which it is required to find the average time for the payment of several sums of money due at different times.

14. Give a problem in which it is required to find the average time for the settlement of an account when there are both debit *and credit items.*

1. What is regarded as the base in all problems in profit and loss? in trade discount? in commission? in brokerage? in insurance? in taxes? in customs or duties? in interest? in bank discount? in true discount? in compound interest?

2. In the example, 8 is 4% of what number? what is the base? percentage? rate? Give the formula for the solution of this problem, using the letters *B*, *P*, and *R*.

3. In the example, What number increased by 20% of itself is 240? what is the amount? Give the formula for solution.

Give the formula for solving each of the following problems:

4. What is the loss on flour that cost \$5 a barrel and sold at a loss of 10%?

5. A merchant sold cloth for 18¢ a yard which cost 16¢. Required the gain per cent.

6. What is the cost of goods which sell for \$80, at a gain of 12½%?

7. What was the cost of a horse which sold for \$400, at a loss of 25%? (*D* = difference.)

8. What is the commission for selling \$500 worth of goods at 1½%?

9. What is the amount of sales when a commission of \$40 is received at 2%?

10. Remittance, \$2000; commission, 2%. Sum invested?

11. Commission, \$80; sum invested, \$4000. Rate?

12. Principal, \$600; rate, 5%; time, 8 mo. Interest?

13. Interest, \$6.70; time, 1½ yr.; principal, \$100. Rate per cent?

14. Rate, 4½%; time, 2 yr. 3 mo.; interest, \$80. Principal?

15. Amount, \$600; discount, 6%; time, 4 mo. Present worth?

16. Face of note \$450; time 4 mo.; rate 6%; proceeds?

17. Proceeds \$380; time 60 da.; rate 7%; face of note?

18. What sum of money put at interest at 5% will amount to \$1200 in 6 mo. 3 da.?

19. In what time will any sum of money put at simple interest double itself at 8%?

1. Show by examples short or convenient ways of adding both in lines and in columns.

2. Show by example how to add two columns at once.

3. Show by examples ways of proving addition.

4. Show by examples a proof of subtraction.

5. Show by examples a practical way of making change.

6. Show by examples a short way of multiplying by 5; by 25; by 125; by $16\frac{2}{3}$; by $33\frac{1}{3}$; by $8\frac{1}{3}$; by $3\frac{1}{3}$; by 11; by any number between 91 and 99; by 99, 999, etc.

7. Show by examples a proof of multiplication.

8. Show by examples how any integer or decimal can be multiplied or divided by 10; by 100; by 1000, etc.

9. Show by examples a short way of dividing by 50; by 25; by $12\frac{1}{2}$; by $33\frac{1}{3}$; by $8\frac{1}{3}$; by $16\frac{2}{3}$; by 125; by $11\frac{1}{9}$.

10. Show by examples a proof of division.

11. Show by examples a short way of adding fractions having 1 for the numerator.

12. Show by examples a short way of multiplying any number containing $\frac{1}{2}$ by itself.

13. Show by examples a short way of multiplying mixed numbers when the fractions are alike; when the whole numbers are alike.

14. Show by examples short ways of calculating interest.

15. Give original problems involving the process of adding, subtracting, multiplying, and dividing denominate numbers. Give the rule in each case.

16. Give practical problems involving the finding of a least common multiple; of a greatest common divisor.

17. Show by examples how notes are discounted at the bank.

18. The bank proceeds of a note are how much less than its true value? Show by examples.

19. Show by example how a stock company is formed.

20. Show by example the difference between stocks and bonds.

21. Show by practical examples what is meant by trade or *commercial* discount.

22. *Give a practical* problem involving partial payments.

Perform the following by the shortest method you can :

1. 87×50.
2. 306×25.
3. $450 \times 12\frac{1}{2}$.
4. $860 \times 33\frac{1}{3}$.
5. 3.84×5.
6. 6.08×125.
7. $40.08 \times 3\frac{1}{3}$.
8. $8.007 \times 16\frac{2}{3}$.
9. 548×11.
10. 4609×11.
11. 96×98.
12. 93×99.
13. 91×97.
14. 91×94.
15. 96×92.
16. 93×94.
17. 996×998.
18. 994×993.
19. 995×997.
20. 991×996.
21. 608×100.
22. 8.03×10.
23. 90.7×1000.
24. 8.37×100.
25. $.086 \times 10$.
26. $786 \div 100$.
27. $8.06 \div 1000$.
28. $.089 \div 10$.
29. $10.07 \div 100$.
30. $7.08 \div 1000$.
31. $706 \div 50$.
32. $896 \div 25$.
33. $464 \div 12\frac{1}{2}$.
34. $8.09 \div 33\frac{1}{3}$.
35. $9.75 \div 125$.
36. $842 \div 16\frac{2}{3}$.
37. $788 \div 8\frac{1}{3}$.
38. $984 \div 33\frac{1}{3}$.
39. $480 \div 11\frac{1}{9}$.
40. $7.008 \div 12\frac{1}{2}$.

41. $8\frac{1}{2} \times 6\frac{1}{2}$; $7\frac{1}{2} \times 9\frac{1}{2}$; $15\frac{1}{2} \times 8\frac{1}{2}$; $12\frac{1}{2} \times 18\frac{1}{2}$; $11\frac{1}{2} \times 17\frac{1}{2}$.
42. $9\frac{1}{2} \times 9\frac{1}{2}$; $8\frac{1}{2} + 8\frac{1}{2}$; $16\frac{1}{2} + 16\frac{1}{2}$; $18\frac{1}{2} \times 18\frac{1}{2}$; $17\frac{1}{2} \times 19\frac{1}{2}$.
43. $(6 \times 18 \times 3 \times 25 \times 4 \times 9 \times 12) \div (8 \times 7 \times 15 \times 6 \times 12 \times 4)$.
44. $(8\frac{1}{2} \times 7 \times 12\frac{1}{2} + 12\frac{1}{2} \times 3\frac{1}{3} \times 7\frac{1}{2}) \div (14 \times 25 \times 2\frac{1}{8} \times 6\frac{1}{4} \times 3\frac{3}{4} \times 1\frac{1}{10})$.
45. $(18 \times 17 \times 3\frac{1}{3} \times 20 \times 80 \times 87\frac{1}{2}) \div (3\frac{1}{3} \times 12 \times 4\frac{1}{8} \times 30 \times 16 \times 6\frac{1}{4})$.
46. $(2.4 \times .25 \times 800 \times .05 \times 50.5) \div (20 \times 1.2 \times .05 \times 10.1 \times 1.01)$.
47. $(400 \times 6.25 \times 7.5 \times 4.50 \times 7.25) \div (25 \times 29 \times 1.025 \times 1.008)$.

48. 25% of 180
49. $33\frac{1}{3}$% of 240
50. 75% of 300
51. $12\frac{1}{2}$% of 520
52. $16\frac{2}{3}$% of 170
53. $37\frac{1}{2}$% of 520
54. 25% of 341
55. 75% of 89
56. $66\frac{2}{3}$% of 810
57. $12\frac{1}{2}$% of 313
58. $6\frac{1}{4}$% of 640
59. $87\frac{1}{2}$% of 560
60. $33\frac{1}{3}$% of 164
61. $16\frac{2}{3}$% of 380
62. $11\frac{1}{9}$% of 200
63. $62\frac{1}{2}$% of 380
64. $37\frac{1}{2}$% of 241
65. $8\frac{1}{3}$% of 384
66. $6\frac{1}{4}$% of 400
67. $83\frac{1}{3}$% of 280

68. 8 is what per cent of 24 ? of 800 ? of 4 ? of .25 ? of 4000 ?
69. 24 is 150% of what number ? 75% ? $\frac{1}{2}$% ? $266\frac{2}{3}$% ?
70. 150% of 800 ? $\frac{1}{2}$% ? $266\frac{2}{3}$% ? $1\frac{1}{4}$% ? 600% ?
71. $\frac{1}{2}$ is 100% of what number ? 450% ? $66\frac{2}{3}$% ? $12\frac{1}{2}$% ?
72. .8 is what per cent of 1 ? of $\frac{1}{4}$? of 2.5 ? of 100 ? of .8 ?
73. 60.30 is 30% of what number ? $1\frac{1}{2}$% of what ? 150% of what ?
74. 80.40 is 10% more than what ? 20% less than what ?
75. $\frac{1}{2}$% of 1000 is what per cent of 600 ? of 2.5 ? of 150 ?
76. What number increased by 20% of itself equals 350 ?
77. What number increased by $133\frac{1}{3}$% of itself equals 175 ?

1. The sum of \$780 is invested at $3\frac{1}{2}\%$, and has yielded \$60.06. How long has the principal been on interest?

2. I paid \$550 a year rent for my house, which I afterwards bought for \$7500. I gave \$4000 cash, and a 4% mortgage for the balance. Allowing money to be worth 4%, and estimating the cost for repairs, etc., each year to be $2\frac{1}{2}\%$ of the cost of the house, how much have I gained a year by buying the house?

3. A house valued at \$6400 was insured for $\frac{3}{4}$ of its value at $\frac{3}{4}\%$ a year. After the house had been insured 10 years, it was damaged by fire to the extent of \$450, which the company paid. How much more or less did the company pay out than they had received in premiums?

4. I sent my agent \$4000 to invest in wheat. After deducting his commission of $2\frac{1}{2}\%$ for buying, and paying \$10.40 for express, etc., he invests the remainder in wheat at 60¢ a bushel. How many bushels did he buy?

5. If 20% is gained on flour when sold at \$7.80 a barrel, what per cent would be gained if it were sold at \$7 a barrel?

6. Sold a horse for \$180, losing 40%. What was the cost?

\$4680. *Boston, March 25, 1893.*

Six months after date I promise to pay John Jones, or order, Four Thousand Six Hundred Eighty Dollars. Value received.

JOHN SMITH.

7. If the above note was presented at the Emporia National Bank May 1, to be discounted, and the rate of discount was 6%, what were the avails? Who indorsed the note? Who may have indorsed it? Who probably received the money at the bank May 1? Who may have received it? When does the bank get its money back? How much? What rate of interest did it get on its money? (Give exact rate per cent.)

8. Which is the better investment, and by how much per cent: Atch. Top. & S. F. 4's bought at $81\frac{1}{2}$, or B. & A. Railroad stock *bought at 175,* the stock paying 7% annually?

1. The tax on a farm is $96.20. The rate of taxation is $14.80 on a thousand, and the assessed valuation is $\frac{3}{4}$ of the supposed real valuation. What is the supposed real valuation of the farm?

2. A certain furniture factory is now able, by the use of machinery, to make the same amount of goods with 77 men as formerly with 110 men. What per cent of the cost of labor has the improved machinery saved?

3. A workman can make 800 two-pound cans per day by hand. By machinery he can make 2400 cans. What decrease per cent in manual labor is here made by machinery? How many hands would be required to turn out 96,000 cans per week by hand? by machinery?

4. How many cu. in. in a quart liquid measure? How much larger is the quart dry measure than the quart liquid measure?

5. A cask containing 2 cu. ft. will hold how many quarts of vinegar? How many quarts of berries?

6. A cistern 6 ft. long, 4 ft. wide, 3 ft. deep will hold how many gallons of water?

7. How deep must a box 6 ft. 3 in. long and 2 ft. wide be made to hold $8\frac{3}{4}$ bushels of cranberries?

8. How high shall I make a box 4 ft. long, 3 ft. wide to hold 200 gallons of water?

9. A bin 10 ft. long and 4 ft. wide must be how high to contain 15 tons of coal, allowing a cubic foot to weigh 90 lb.?

10. If a cubic foot of ice weighs 57 lb., how many tons of ice can be stored in an ice-house which is 240 ft. long, 40 ft. high, and 3 ft. wide?

11. British gold coin consists of 11 parts gold and 1 part copper. How many pennyweights of pure gold in an English sovereign which weighs 123,374 grains troy?

12. If the alloy used in making a certain kind of brass consists of two parts by weight of copper to 1 part of zinc, how much copper is used to make 100 lb. of brass? How much if the ratio of copper to zinc is 3 to 1? How much if 4 to 1? How much if 4 to 5?

The following table gives the estimated number of persons in the world speaking the seven principal European languages in 1801 and in 1890:

Languages.	Number of Persons Spoken by.		Percentage of Increase in Eighty-nine Years.	Percentage of the whole.	
	1801.	1890.		1801.	1890.
English	20,520,000	111,100,000	.....	.....	.....
French	31,450,000	51,200,000	.....	.....	.....
German	30,320,000	75,200,000	.....	.....	.....
Italian	15,070,000	33,400,000	.....	.....	.....
Spanish	26,190,000	42,800,000	.....	.....	.....
Portuguese	7,480,000	13,000,000	.....	.....	.....
Russian	30,770,000	75,000,000	.....	.....	.....
Total	.....	.....	.....	.....	.....

1. Fill the blanks in the above table, carrying out figures in percentage columns to tenths of one per cent.

2. An insolvent debtor has liabilities of $60,000 and assets worth $25,000. How much will a creditor obtain to whom he owes $6600?

3. A traveller on a train going at the rate of 40 miles an hour counts 96 telegraph poles in 4 minutes. How many poles are there to the mile?

4. The longitude of Sandy Hook is 74° W.; that of Queenstown 8° 7′ 30″ W. The "City of New York" makes the run between these points in 5 days 20 hours. If she passed Sandy Hook at 5 P.M. June 15th, when did she arrive off Queenstown?

5. A room is 30 ft. long, 24 ft. wide, and 15 ft. high. What will it cost to plaster it at 12¢ a square yard, allowing for 4 doors each 8×4 ft., 5 windows each $9 \times 3\frac{1}{2}$ ft., and a wainscot 28 in. high?

6. I own a lot whose four sides measure respectively 40, 50, 38, and 48 rods. Draw plan and find approximate area.

7. If 1000 laths cover 39 sq. ft. of surface, how many laths will be required for your school-room? how many for your sitting-room *at home*?

1. A grocer gained 25% by selling 20 lb. of sugar for a dollar. How much per cent will he gain by selling 25 lb. for a dollar?

2. A merchant secures goods at 25% and 33⅓% off list prices, and sells at a discount of 20% from same list. What was the gain per cent?

3. A stack of ice containing 10,000 tons is bought for 90¢ a ton. The cost of handling is 10¢ a ton, and freight rate to market is 3½¢ a hundred. For how much may I contract to deliver it and gain 50% on the sum invested?

4. In example 3, if the investment was made February 24, 1890, and amount due on contract was paid November 30, 1890, money being worth 7½%, what was my real gain?

5. What sum of money put on interest at 4% will yield an income of $500 semi-annually?

6. A corporation sends its agent $4590, including commission at 2%, with which to buy sugar. If 75 tons are obtained, what is the cost per pound?

7. My tax bill is $162.80, which includes a poll tax of $2. The rate of taxation being $13.40 on $1000, for how much am I assessed?

8. A shipment of hides invoiced at $48,000, and insured at ⅞% on ¾ its value, was lost at sea. What was the loss to the owner? to the insurance company?

9. What is the duty at 20% on 400 boxes of raisins, each weighing 40 lb., tare 5%, invoice price 8¢ a pound?

10. What is the cost of 120 shares Mexican Central stock at 20¼; 12 shares Old Colony at 166¾; and 25 shares Boston & Albany at 202? What sum would be gained or lost by selling all at par? Brokerage ⅛%.

11. Which is the better investment, and by how much per cent: Chicago & Southern Michigan, which pays a dividend of 4%, costing 60, or Connecticut River, which pays a dividend of 8%, and costs 200?

12. A man bought stock at 122½ and received $400 when a 2½% dividend was made. What sum was invested?

1. October 31, 1890, I accepted in payment for a house a note for $3000 payable in 6 months without interest. I immediately had it discounted at 6½%. What did I really obtain for the house?

2. If $500 is procured on a 90-days' note not bearing interest, for how much was the note written, discount rate 5%?

3. Sugar-cane contains about 15% of sugar, beet root 10%, and wheaten flour 8%. On the supposition that one-half of the sugar is lost in the process of manufacture, how many pounds of each will be required to make 200 lb. of sugar?

4. If a cubic inch of water weighs 253.7 grains, and the specific gravity of air is .001292, what will the air of your school-room weigh?

5. What degree upon a centigrade thermometer corresponds to 32° Fahrenheit? 212° F.? 60° F.? 80° F.? When the centigrade marks 20°, what will the Fahrenheit mark?

6. A bar of gold is 20 cm long, 40 mm wide, and 40 mm thick. What is its weight, the specific gravity of gold being 19.36?

7. How many prescriptions of 20 grains each can be put up from 2 lb. 6 ℥ 4 ℨ?

8. If coal gives out 50% more heat than an equal weight of wood, how many cords of wood will give out as much heat as a ton of coal, reckoning the wood to weigh 52 lb. to the cubic foot?

9. The total number of immigrants into the United States during the ten years ending 1890 was 5,238,728. Of these 1,452,952 came from Germany, 649,052 from England, 655,381 from Ireland, 560,483 from Norway and Sweden. What per cent of the whole came from each of these countries?

10. The value of United States exports to foreign countries for the year ending June 30, 1893, was $847,665,194, and of imports from foreign countries was $866,400,922. The value of exports to England was $361,410,055, and of imports was $154,281,905. By what per cent was the excess of imports more than exports? The value of exports to England was what per cent of the entire exports? The value of imports from England was what per cent *of the entire imports*?

SECTION II.

ALGEBRAIC EXERCISES.

1. $8+7$ expresses what operation? Express in the same way the sum of the numbers a and b; of c and d; of x and y.

2. $8-3$ expresses what? Express in the same way the subtraction of the number b from the number a; d from c; y from x.

3. In each of the expressions 4 twos, 4 times 8, 4 a's, 4 times b, what number is the multiplicand? What the multiplier? What is meant by the expression $6a$? $8b$? $20c$? ab? xy?

4. Express in the form of a fraction $20 \div 4$. In the same way express: $a \div b$; $c \div d$; $x \div y$; $2 \div a$; $8 \div d$.

5. If a represents the number of crayons in one box and b represents the number of crayons in another box, how will you express the number of crayons in both boxes?

6. John has c apples and James has d apples. Express the number of apples that both have.

7. If a represents the number of apples that William had, and b the number he gave away, how can you express the number he had left?

8. If each of 4 boys has x cents, what expression would represent the number that all have?

9. If x represents the number of cows that A has, and B has four times as many, what would represent the number of cows that B has? What the number that both have?

10. Shorten the expressions: $x+x+x$; $3x+2x+x$.

11. If a man has 20 horses, and x represents the number he sells at one time, and $2x$ represents the number he sells at another time, how will you express the whole number he sells? How the number he had left?

1. If x represents the number of cents William has, and John has one-fourth as many, what represents the number that John has? What the number that both have?

2. Represent the cost of 8 pounds of sugar at a cents a pound.

3. Represent the cost of 1 doz. eggs, if 4 doz. cost a cents.

4. Represent the cost of $\frac{1}{2}$ doz. eggs, if 1 doz. cost a cents.

5. Represent the cost of $\frac{3}{4}$ of a pound of meat at a cents a pound.

6. Shorten the expressions: $2a+6a+10+2a+4$; $7x+3y+2+8+4y+3x$; $9a+4b+7a+3b+6+2a+9$.

7. Read the equations: $10=7+3$; $20=24-4$; $x=a+4$; $x=a+b$; $3x+2y=40$.

8. Find the value of x and y in the following equations, letting $a=8$ and $b=6$: $x=2a+4$; $x=9a+2b$; $y=7a+6b$; $x=4a-3b$; $2x=2a+2b$.

9. Find the value of x and y in the following equations:

$$12x=20+4; \qquad 4x+3x=63; \qquad y+3y=36.$$

10. Express 4 times x; $\frac{1}{2}$ of x; $\frac{3}{4}$ of x; $1\frac{1}{2}$ times x; $6\frac{2}{3}$ times x; $x\div y$; $3x\div 4$.

Find the value of x in the following equations:

11. $\frac{x}{4}=12$; $x+\frac{x}{2}=24$; $\frac{3x}{4}=18$.

12. $\frac{6x}{8}=6$; $\frac{3x}{4}+\frac{x}{4}=16$; $\frac{5x}{6}-\frac{x}{3}=14$.

13. $\frac{3x}{10}+\frac{x}{5}=25$; $\frac{8x}{12}-\frac{x}{6}=24$; $\frac{5x}{9}-\frac{x}{3}=20$.

14. $x+\frac{3x}{4}-\frac{x}{4}=12$; $\frac{5x}{8}+\frac{3x}{4}=22$; $x-\frac{3x}{10}=14$.

15. $\frac{114}{12}-\frac{x}{3}=12$; $\frac{5x}{6}+\frac{3x}{4}=9\frac{1}{2}$; $7x+\frac{x}{2}+\frac{x}{4}=31$.

16. How can you divide 18 cents between John and James so that John will have twice as many cents as James? (Let x represent *the number of cents* that James has.)

1. A man bought a horse and a cow for \$360, paying 4 times as much for the horse as for the cow. What was the cost of each?

2. A has twice as much money as B, and C has as much as A and B together. They all have \$300. How much money has each person?

3. Robert has twice as many marbles as William, and Thomas has three times as many as Robert. They all have 180 marbles. How many has each?

4. If I buy some oranges at $3¢$ apiece, and the same number of bananas at $2¢$ apiece, how many of each can I buy for $45¢$? (Let x = number of oranges or bananas. What will represent the number of cents that the oranges cost? the bananas? all?)

5. The difference between nine times a certain number and sixteen times that number is 84. What is the number?

6. A man is twice as old as his son. If the difference in their ages is 24 years, what is the age of each?

7. Three kinds of tea were put together in equal quantities, the prices per pound being $50¢$, $60¢$, and $70¢$ respectively. How many pounds of each are there in the mixture that is worth \$14.40?

8. A mixture of Java coffee worth $28¢$ a pound, and Mocha coffee worth $30¢$ a pound, is worth in all \$4.56. There is three times as much Java as Mocha. How many pounds of each in the mixture?

9. A, B, and C entered into a partnership with a capital of \$10,000. A put in twice as much as B, and C put in $\frac{1}{3}$ as much as B. What was the capital of each?

10. If $\frac{1}{2}$ of A's money is worth $\frac{3}{4}$ of B's, and B has \$400, what has A?

Simplify:

11. $(x+5)+(x+8)$; $(x-9)+(x-8)$; $(x-9)+(x-12)$.
12. $(3x+10)-(2x+5)$; $(4x-20)-(2x-8)$; $(12x-30)-(6x-20)$.
13. $(2a+3b)+(3a+2b)$; $(5a-2b)+(2a-3b)$; $(8a-6b)+(3a-4b)$.
14. $(8a-4b)-(6a-2b)$; $(9a-6b)-(2a-5b)$; $(12a-18b)-(9a-14b)$.
15. $(x+2)\times 4$; $(3x+8)\times 8$; $(4x-3)\times 4$; $(2x-8)\times 6$.

Simplify :

1. $(4x+4)\div 2$; $(9x-3)\div 3$; $(12x-48)\div 12$; $(36x+24)\div 12$.
2. $\frac{1}{3}$ of $(18x-9)$; $\frac{3}{4}$ of $(16x-4)$; $\frac{4}{5}$ of $(20x+15)$.
3. $\frac{1}{2}$ of $(x+3)$ (express the division); $\frac{1}{3}$ of $(x-1)$.
4. $\frac{1}{5}$ of $(3x-1)$; $\frac{3}{4}$ of $(x-3)$; $\frac{2}{5}$ of $(5x-8)$; $\frac{7}{8}$ of $(5x+18)$.

5. If $x=8$, what will $x+2$ equal? Write: $x+2=8+2$; $x+x=8+8$. What is the effect of adding the same or equal quantities to equal quantities? Illustrate by problem.

6. If $x+4=20$, what is the value of x? If $x=16$, what is $x-5$ equal to? What is the effect of subtracting the same or equal quantities from equal quantities? Illustrate by problems.

7. Find the value of x in the equation $3x=24$. Find the value of $6x$. What is the effect of multiplying or dividing equal quantities by the same or equal quantities? Illustrate by problems.

8. If $x=18$ and $y=18$, what can you say of the relative values of x and y? What can you say of two quantities that are each equal to a third quantity? Illustrate by problems.

Find the value of x in each of the following equations :

9. $x-6=12$; $x+8=30$; $x-16=40$; $x+18=80$.
10. $6x=48$; $3x+6=12$; $4x-8=16$; $9x+14=68$.
11. $\frac{x}{3}=12$; $\frac{x}{8}=6$; $\frac{2x}{3}=6$; $\frac{3x}{4}=12$; $\frac{5x}{8}=10$.
12. $\frac{x}{4}+2=8$; $\frac{x}{3}-6=4$; $\frac{3x}{4}+2=8$; $\frac{3x}{2}-3=12$.
13. $\frac{4x}{5}+20=40$; $\frac{5x}{12}-16=44$; $\frac{8x}{20}+18=22$; $\frac{3x}{14}-31=73$.
14. $\frac{2x}{3}-18=10$; $\frac{x}{2}+x=9$; $\frac{3x}{4}+x=7$; $\frac{2x}{3}+\frac{x}{3}=9$.
15. $\frac{x}{2}+\frac{x}{4}=6$; $\frac{x}{3}+\frac{x}{2}=10$; $x-\frac{3x}{4}=12$; $\frac{2x}{3}-\frac{x}{2}=6$.
16. $\frac{3x}{4}-\frac{x}{3}=5$; $\frac{4x}{6}-\frac{x}{3}+2=6$; $\frac{8x}{9}+\frac{2x}{3}-3=1\frac{2}{3}$.

Find the value of x in each of the following equations :

1. $2x = x + 8$; $6x = 2x + 12$; $4x = 8 + 2x$; $9x - 8 = 8x$.
2. $3x = 8 - x$; $8 + 3x = 4x$; $4x + 6 = 7x$; $20 - 3x = 2x$.
3. $18 - 2x = 4x$; $5x - 30 = 2x$; $6x = 50 - 4x$; $8x - 8 = 3x$.
4. $2x + 4 = x + 12$; $6x + 3 = 2x + 15$; $3x + 20 = 6x + 2$.
5. $6x + 18 = 4x + 36$; $8x - 12 = 2x + 24$; $9x + 10 = 12x - 20$.
6. $6x - 18 = 0$; $8x - 56 = 0$; $0 = 24 - 2x$; $0 = 6x - 24$.
7. $x + 9 = \frac{2x}{3} + 12$; $x + 8 = \frac{3x}{4} + 13$; $2x - 19 = 14 + \frac{5x}{8}$.
8. $\frac{3x}{4} - \frac{x}{2} = 15 - x$; $4x - \frac{3x}{5} = 3x + 8$; $40 - \frac{4x}{9} = \frac{x}{3} + 19$.
9. $\frac{x+1}{2} = 3$; $\frac{x+4}{6} = 2$; $\frac{2x+6}{8} = 7$; $3x = 9x - 7x + 27$.
10. $3x + 1 = \frac{4x-2}{2} + 8$; $\frac{3x}{4} - 1 = \frac{3x+12}{6}$; $\frac{2x+6}{3} = \frac{4x+36}{6}$.

Remove parentheses from the following expressions :

11. $20 - (6 + 4)$; $18 - (10 + 3)$; $2x - (x + 1)$.
12. $4x - (2x + 8)$; $8x - (18 + 4)$; $6x - (18 + 3x)$.
13. $12 - (8 - 4)$; $9x - (2x - 1)$; $5x - (20 + x)$.
14. $5x - (2x + 6)$; $20x - (12x + 16)$; $7x - (8x - 6)$.
15. $16x - (18 + 4x)$; $24 - (16 - 8x)$; $12x - (7x + 12)$.

Find the value of x in the following equations :

16. $x - \frac{x+4}{2} = 2$; $x - \frac{x+3}{3} = 3$; $x - \frac{2x-6}{8} = 12$.
17. $2x - \frac{x+15}{9} = 21$; $3x - \frac{2x+8}{3} = 21$; $8x - \frac{3x-4}{4} = 30$.
18. $\frac{x+6}{2} - \frac{x+3}{3} = 4$; $\frac{2x+2}{2} - \frac{3x-8}{4} = 5$; $\frac{8x+9}{9} - \frac{60-2x}{6} = 2$.
19. $\frac{2x-6}{6} - \frac{x-12}{4} = 3$; $\frac{20+x}{2} - \frac{x-10}{8} = 18$; $\frac{60+2x}{12} - \frac{3x-45}{60} = 9\frac{1}{4}$.
20. $\frac{4x-8}{3} - \frac{2x-4}{6} = 6$; $\frac{x}{2} - \frac{x-12}{3} = 8$; $\frac{30+x}{2} - \frac{2x-20}{6} = 20\frac{1}{2}$.

1. If a bushel of wheat costs 12 cents more than a bushel of corn, and a bushel of both costs $1.48, what is the cost of each?

2. James is 6 years less than twice the age of his sister. Their united ages amount to 42 years. How old is each?

3. John had twice as many cents as Robert. After John had spent 6 cents and Robert had spent 4 cents, they both together had 26 cents. How many had each at first?

4. Divide the number 82 into three such parts that the first may exceed the second by 6, and the second exceed the third by 8.

5. Divide the number 60 into three such parts that the first and second numbers shall be equal, and that the third number shall be 12 less than the sum of the first two.

6. How many pounds of five-cent sugar and of seven-cent sugar shall be put together to make a mixture of 20 pounds, worth $1.16?

7. In a flight of stairs consisting of eight steps, each step is 1 inch higher than the one below it. If the height of the stairs is 7 ft., how high is each step?

8. Divide 40 marbles between two boys so that one has $\frac{2}{3}$ as many as the other.

9. What number increased by one-eighth of itself amounts to 27?

10. $\frac{2}{3}$ of a number added to $\frac{3}{4}$ of that number is equal to 68. What is the number?

11. $2\frac{5}{8}$ times a certain number increased by 12 is equal to 117. What is the number?

12. After Robert had spent $\frac{3}{4}$ of his money, and Ralph had spent $\frac{2}{3}$ of his money, they had 24 cents apiece. How much had each at first?

13. A farmer sold $\frac{1}{6}$ of his farm to one man and $\frac{1}{4}$ of it to another man. If the difference between the two parts sold was 15 acres, how much land did the farm contain originally?

14. 3 years taken from half the age of Mary is equal to $\frac{2}{5}$ of the age of Julia. If the sum of their ages is 27, what is Mary's age?

15. Two boys divided some apples so that one had 4 more than $\frac{1}{3}$ of them, and the other had 1 less than half of them. How many *had each boy?*

1. A father is now $2\frac{1}{2}$ times the age of his son, but 9 years from now he will be twice as old. How old is each now?

2. Three children inherit property, sharing it as follows: The first has \$100 more than $\frac{1}{8}$ of it; the second has \$200 less than $\frac{4}{11}$ of it; and the third has $\frac{5}{11}$ of it. How much has each?

3. A has a certain sum of money, B has 50% more than A, and C has 50% more than B. If they all have \$19,000, how much has each man?

4. A man spent one day \$50, and the next day he spent 50% of what then remained. At the end of the second day he found he had $\frac{2}{5}$ of what he had at first. What had he at first?

5. How much tea at 60¢ a pound must be mixed with 20 lb. of tea at 85¢ a pound to have a mixture worth 75¢ a pound?

6. What number diminished by $\frac{3}{4}$ of itself will leave 200?

7. What number increased by $\frac{5}{8}$ of itself will amount to 80?

8. What number increased by 25% of itself will amount to 800?

9. A boy gave away 25% of his marbles and found he had 30 marbles left. How many had he at first?

10. By selling flour at \$6 a barrel, a merchant gains 20%. What is the cost?

11. By selling cloth at a gain of 25%, 2¢ a yard is gained. What is the cost?

12. What sum of money put at interest 6 months at 5% will amount to \$820?

13. What sum of money put at interest at 5% will give the same income as \$6000 put at interest at 4%?

14. At what rate of interest will \$8000 yield the same income as \$6000 at 6%?

15. What is the interest of \$5000 for 4 years at $4\frac{1}{2}$%?

16. Find the interest of \$800 for 1 yr. 9 mo. at 5%.

17. A and B invested the same amount of money in business. At the end of three years it was found that A's capital was \$500 less than what he had at first, that B's capital was \$1500 less than twice his original capital, and that the combined capital of both had increased 25%. What was the original investment of each?

Relative Series.

Absolute Series.

etc. $-\dot{4}$ $-\dot{3}$ $-\dot{2}$ $-\dot{1}$ $\dot{0}$ $+\dot{1}$ $+\dot{2}$ $+\dot{3}$ $+\dot{4}$ etc.

etc. $-4a$ $-3a$ $-2a$ $-1a$ 0 $+1a$ $+2a$ $+3a$ $+4a$ etc.

Negative Quantities. Positive Quantities.

Add :

1.	2.	3.
$+4a$	$-3a$	$+4a$
$+3a$	$-2a$	$-3a$
$+2a$	$-a$	$+2a$

Subtract :

4.	5.	6.
$+4a$	$+4a$	$-4a$
$+3a$	$-2a$	$+3a$

7. Show by examples that the subtraction of any quantity gives the same result as the addition of the same quantity with the opposite sign.

8. $(+3ab)+(+2ab)$; $(+4ab)+(-3ab)$; $(-5ab)+(-3ab)$.

9. $(+5ab)-(+2ab)$; $(-5ab)-(-2ab)$; $(+5ab)-(-2ab)$.

10. Represent $+2a$ taken 4 times additively; $(+2a)\times(+4)=?$

11. Represent $+2a$ taken b times additively; $(+2a)\times(+b)=?$

12. Represent $-2a$ taken b times additively; $(-2a)\times(+b)=?$

13. Represent $+2a$ taken b times subtractively; $(+2a)\times(-b)=?$

14. Represent $-2a$ taken b times subtractively: $(-2a)\times(-b)=?$

15. Divide $(+2a)\times(+b)$ by $+2a$; $+2ab$ by $+b$; $-2ab$ by $-2a$; $-2ab$ by $+b$; $-2ab$ by $+2a$.

16. From $8a-6b+4b-6c+2d-e-3f+8g-h+6i-4j$
Take $3a-3b-2b+8c+6d-3e+2f-3g+4h-3i-3j$

Add :

17.	18.	19.
$16-4+7$	$3a-4b+2c$	$7a+8b-14c$
$-4+8-3$	$-4a+6b+3c$	$6a-9b-3c$
$8-12+6$	$8a-9b-8c$	$-18a+6b+18c$

Subtract :

20.	21.	22.
$18-6+12$	$12a+4b-7c$	$15a-6b+10c$
$-12-8+6$	$5a-9b+12c$	$-7a+9b-4c$

Reduce each of the following expressions to simplest form :

1. $4 \times 4 = 4^2$; $a \times a$; $c \times c \times c$; $2b \times b$.
2. $(-a) \times (-a)$; $(+a) \times (-a)$; $(+2a) \times (+a)$; $(+3b) \times (-b)$.
3. $(a^2) \times (a)$; $(ab) \times (a)$; $(a^2b) \times (a)$; $(a^2b) \times (-a)$.
4. $(a^2b) \times (b)$; $(-a^2b) \times (a)$; $(ab^2) \times (-b)$; $(abc) \times (a)$.
5. $(ab) \times (cd)$; $(2a^2b) \times (b)$; $(2a^2b) \times (-a)$; $(a^2b^2) \times (b^2)$.
6. $(ab^2c) \times (2a)$; $(2a^2) \times (-a^2)$; $(4a^2b) \times (2c)$; $(6a^3b^2) \times (ab)$.
7. $(6a^2) \times (-2a^3)$; $(4a^2b) \times (-m)$; $(3b^2c^3) \times (2a^3)$; $(-a^3) \times (a^2)$.
8. $(-2a)^3 \times (-a^2)$; $(ab) \times (ab)$; $(ab)^2 \times (ab)$; $(a^2b)^2 \times (a)$.
9. $(ab^2)^3 \times (3a)^2$; $(4a^2b^2)^2 \times (-2a)^2$; $(-3a^2b)^3 \times (-2ab^3)$.
10. $(4ab) \div (4a)$; $(8a^2b) \div (8b)$; $(6ab) \div (2a)$; $(6ab) \div (-2a)$.
11. $(8ab) \div (-4a)$; $(3ab) \div (ab)$; $(8abc) \div (4c)$; $(6ac) \div (3a)$.
12. $(a^4) \div (a^3)$; $(4a^4) \div (2a^2)$; $(6a^3b^2) \div (3a^2b)$; $(12a^5b^2) \div (ab)$.
13. $(9a^3b^2) \div (3a^2b)$; $(15a^2b^3c^4) \div (3a^2c^3)$; $(18a^2b^3) \div (-6ab^2)$.
14. $(ab) \div (cd)$; $(6a^2b) \div (ab)$; $(6ab) \div (ac)$; $(3a^2b) \div (-b)$.
15. $(8ab^3) \div (ac)$; $(9ab^2) \div (3ac^2)$; $(10ab) \div (5ac^2)$; $(6a^3b^3) \div (-a^2b^2)$.
16. $(12a^2b^2) \div (6c^2)$; $(-3a^2)^2 \div (-a^2)$; $(-18a^4b^6) \div (-a^3b^4)$; $(3b^2c) \div (4d^2)$.
17. $(4a^2b) + (2a^2b)$; $(8ab^2) + (-6ab^2)$; $(-3abc) - (-2abc)$.
18. $(8a) + (-6b)$; $(5a^2b^2) - (2a^2c)$; $(7a^2) + (-4a^2) - (2a^2)$.
19. $(4a^2b) - (-3ab^2)$; $(12ab) - (-6ab^2)$; $(7a^5) + (3a^4) - (2a^3)$.
20. $(-ab) - (-b)$; $(4ab^2) - (3a^2b)$; $(5a^5b^2) - (-5b^2)$.
21. Add : $x + y$; $2x + 2y$; $3x - y$; $-4x - 4y$.
22. Add : $3x^2 + x + y$; $x + y$; $3x^2 - 2y$; $4x + 3y$.
23. Add : $4xy^2 + 4x^2$; $3x^2 + 2x^2y$; $-2xy^2 - 6x^2y$.
24. Add : $x^2 + 4xy - y^2$; $4x^2 + 2xy + 3y^2$; $4y^2 - 6xy$.
25. Add : $8x^3y^2 - 6x^2y^3 + 3x$; $4x^3y^2 + 18$; $8x^2y^3 - 16x$.
26. From $8x^2y + 3xy^2 - 6x - 8$ take $-4xy^2 \times 8x - 9$.
27. From $6x^3 - 8y^2 + 7x^2y - 6xy^2$ take $-8x^3 - 6x^2y + y^2$.
28. From $3x^3y^4 - 4y^5$ take $-2x^4 + 3x^3y^4 - 6x^2y^3$.
29. From $4x^4 + 3y^4$ take $2x^4 - 3x^3y + 5x^2y^2 - 3xy^2 + y^4$.
30. From $\frac{x+y}{4} + \frac{x-y}{2}$ take $(2x + 3y) \times \frac{1}{3}$.
31. From $x + \frac{y}{2} + 4$ take $\frac{x}{4} + y - 8$.

1. Multiply $a+b$ by x; by y; by $-x$; by $-y$.
2. Multiply $a+b$ by $x+y$; by $x-y$; by $-x-y$.
3. Multiply $a-b$ by $x+y$; by $x-y$; by $-x-y$.
4. Multiply $a+1$ by $a+1$; by $a-1$; by $1-a$.
5. Multiply $3a+2b-4c$ by $2x$; by $-2xy$; by $3a$.
6. Multiply $2ax+3a^2-4$ by $2a$; by $2ax$; by $2a-1$.
7. Multiply $4a^2b+2ab^2+b^3$ by a; by $a-1$; by $1-a$.
8. Multiply $a+b+c$ by $a+b$; by $a-b$; by $a+b+c$.
9. Multiply $2x^2y-4xy^2+y^3$ by x; by $x+1$; by $x-1$.
10. Multiply $x+y$ by $x+y$; by $x-y$; by x^2+y^2; by x^2-y^2.
11. Multiply x^2y+xy by $x+y$; by $xy-1$; by x^2y; by xy^2.
12. Multiply together $x+1$, $x-2$, $x+3$, $x-4$.
13. $(x+y)^2=?$ $(x-y)^2=?$ $(x+1)^2=?$
14. $(x-1)^2=?$ $(2x+2y)^2=?$ $(2x-2y)^2=?$
15. $(a+b)^3=?$ $(a-b)^3=?$ $(a+1)^3=?$
16. $(ab+cd)^2=?$ $(ab+1)^2=?$ $(x^2+y^2)=?$
17. $(a+1)\times(a-1)+(a+1)\times(a+1)=?$
18. $(x+y)\times(x+y)+(x-y)\times(x-y)=?$
19. $(a^2+2ab+b^2)\times(a+b)-(a^2+2ab+b^2)\times(a-b)=?$
20. Divide $ab+ac+ad$ by a; by ab; by ac; by ad.
21. Divide $6a^2b+12ab^2+18ab$ by ab; by $6ab$; by $6b$.
22. Divide $(a+b)^3$ by $(a+b)^3$; by $(a+b)^2$; by $(a+b)$.
23. Divide $(a-b)^4$ by $(a-b)^3$; by $(a-b)^2$; by $(a-b)$.
24. Divide $a^2+2ab+b^2$ by $a+b$; by $(a+b)^2$.
25. Divide (a^2-b^2) by $a+b$; by $a-b$.
26. Divide $a^3+3a^2b+3ab^2+b^3$ by $a+b$; by $a^2+2ab+b^2$.
27. Divide $4x^2-1$ by $2x-1$; by $2x+1$.
28. Divide $4x^2-4x+1$ by $1-2x$; by $2x-1$.
29. Divide $ab-ac$ by a; by $-a$; by $b-c$.
30. Divide $(ac-bc)+(ad-bd)$ by $a-b$.
31. Divide $6+17x-3x^2-20x^3$ by $3+4x$.
32. Divide a^4-b^4 by a^2+b^2; by a^2-b^2.
33. Divide $4x^2+3xy+8x+3y+4$ by $x+1$.
34. Divide $6x^2+2xy+2x-4y^2+2y$ by $2x+2y$.
35. *Divide* $6x^2+4xy-12x-8y$ by $2x-4$.

Remove brackets and simplify :

1. $20+(4+2)$; $a+(b+c)$; $2a+(3a+a)$.
2. $20+(8-3)$; $a+(b-c)$; $6a+(4a-2a)$.
3. $80-[60+10-40-(20-10)+20]+30-(60+20)$.
4. $a-b-[c-d-(2c+d-3b)+(2d-g)]$.
5. $a-x^2-(x^2+a)-[x^2-(a+x)+a]$.
6. $9x^2-[3y^2-(x+y^2-z)+2x^2-(3x+z)-3y^2]$.
7. $(4x+1)+(4x-1)+6x^2+1-(2x^2+1)+\frac{6x^2}{4}$.
8. $(3+x)-(18-5x^2)+(x+y)-(x-y)$.
9. $(x^3+y^3)-(x-y^3)+(x^3+1)-(x-1)-(x^3-y^3)$.
10. Raise to the second power : x; x^2; y^3; xy; x^2y; xyz; xy^2z; $x+y$; $x-y$; $x+1$; x^2+y^2; $3a+2b$.
11. What is the square of the sum of two quantities? What is the square of the difference?
12. Raise to the third power : x^2; xy; xyz; x^2yz; $x+y$; $x-y$; $2x+2$; $3x-1$; $4a+2b$.
13. What is the cube of the sum of two quantities? What is the cube of the difference?
14. $(2x+3y)^2=?$ $(3x-4)^2=?$ $(x+2y)^2=?$ $(3x-6y)^2=?$
15. $(1+x)^3=?$ $(2x+y)^3=?$ $(3x-2)^3=?$ $(x-3y)^3=?$
16. $(4x+a)^2=?$ $(2x-3)^3=?$ $(4a+3b)^2=?$ $(2a-5)^3=?$
17. $(4x+3y)^2-(4x-3y)^2+(2x+y)^2-(2x-y)^2$.
18. $(3a+6b)^2-(2a+3b)^2+(4a-2b)^2-(6a+b)^2$.
19. $(a+b)^3-(a-b)^3+(2a+2b)^3-(2a-2b)^3$.
20. $(x+1)^3+(1+2x)^3-(2x+y)^3-(2x-2y)^3$.

Separate into factors :

21. $4a^2$; $8a^3$; $16a^2b$; $12a^3b^2c$; $(a+1)^2m$.
22. $ax+ay$; a^2x-ay; $a^3x^3-x^3$; a^2b-a^2x.
23. a^2-b^2; $a^2+2ab+b^2$; x^2+2x+1; x^2-1.
24. $a^3+3a^2b+3ab^2+b^3$; a^3-b^3; a^3+b^3.
25. $4a^2+12ab+9b^2$; $9x^2+54x+81$; $9x^2-81$.
26. $8ab^3-4a^2b^2+12a^3b$; $16a^3b^2-8a^4b^4+16a^4b^2$.
27. $4x^2+2x+4xy+2y$; $8x+16y+4xy+8y^2$.

Express the value of x in the following equations :

1. $x+y=8$; $2x+z=10$; $x-2y=12$; $\frac{x}{2}=y+z$.

2. $\frac{x+2}{4}=y$; $2x+y-\frac{x}{2}+\frac{y}{4}=12$; $\frac{3x+2y}{8}=\frac{x+18}{4}$.

3. $x-a=b+8$; $3x-a+b=\frac{2a}{4}$; $2x+\frac{2a}{9}=\frac{36+9}{9}$.

4. $\frac{x+a}{8}+\frac{b+6}{4}=12$; $\frac{3x+a}{5}+\frac{b}{2}=\frac{8a}{10}$; $\frac{x}{a}+b=4-2b$.

Express the value of x and of y in the following equations :

5. $x+y=18$; $x-y=2$; $2x+y=28$; $2x-y=12$.

6. $3x+2y=32$; $4x-3y=20$; $\frac{3x}{4}+y=10$; $x+\frac{y}{2}=10$.

7. $4x-3y=\frac{a}{4}$; $\frac{3x}{9}+\frac{4y}{3}=\frac{a}{3}$; $\frac{2x-4y}{3}+a=\frac{b-1}{6}$.

8. $\frac{3x+2y}{8}=\frac{x-y}{4}$; $\frac{3x}{4}-\frac{2y}{3}=\frac{x-y}{6}$; $\frac{4x}{a}+8=\frac{3y+8}{3}$.

9. If from the equation $3x+2y=18$ you take the equation $2x+2y=14$, what remains? If to the equation $2x+2y=14$ you add $3x-2y=6$, what is the sum?

10. Show by adding the two equations $x+y=9$ and $3x-y=11$ how you can find the value of x. What is the value of y ?

11. Show by subtraction in the equations $x+y=9$ and $3x+y=19$ how you can find the value of x.

12. Having two equations with two unknown quantities in each, how, by addition or subtraction, may the value of one of the unknown quantities be found?

Find by addition or subtraction the value of x and y in the following equations :

13. $\begin{cases} x+4y=16. \\ x-2y=4. \end{cases}$

14. $\begin{cases} x+y=11. \\ 2x+3y=28. \end{cases}$

15. $\begin{cases} 2x+3y=29. \\ x+2y=17. \end{cases}$

16. $\begin{cases} 3x+2y=21. \\ 9x-4y=3. \end{cases}$

17. $\begin{cases} 2x+6y=30. \\ x-3y=3. \end{cases}$

18. $\begin{cases} 5x+4y=55. \\ 3x+3y=39. \end{cases}$

1. In each of the two equations $x - y = 2$ and $x + 3y = 26$ express the value of x. Apply the principle: Quantities that are equal to the same quantity are equal to each other. What is the equation? What is the value of y? By substitution find the value of x.

2. By comparison of equations, as in the last exercise, find the value of x in the equations $x + y = 8$ and $2x + y = 14$. What is the value of y?

By comparison find the value of x and of y in the following equations:

3. $\begin{cases} x + y = 9. \\ 3x + y = 17. \end{cases}$ **4.** $\begin{cases} 4x + y = 14. \\ 3x + y = 11. \end{cases}$ **5.** $\begin{cases} 2x + 3y = 24. \\ 3x + y = 22. \end{cases}$

6. $\begin{cases} 2x - y = 4. \\ 3x + 2y = 27. \end{cases}$ **7.** $\begin{cases} 2x + 3y = 25. \\ x - 2y = 2. \end{cases}$ **8.** $\begin{cases} 3x + 2y = 46. \\ 4x - 3y = 16. \end{cases}$

9. $\begin{cases} 2x + 5y = 53. \\ 4x - 4y = 8. \end{cases}$ **10.** $\begin{cases} 3x - 6y = 12. \\ 4x + 3y = 60. \end{cases}$ **11.** $\begin{cases} 2x + 4y = 66. \\ 8x - 3y = 36. \end{cases}$

12. $\begin{cases} 5x + 2y = 74. \\ 6x + 3y = 96. \end{cases}$ **13.** $\begin{cases} 3x + 2y = 63. \\ 8x - 6y = 66. \end{cases}$ **14.** $\begin{cases} 4x + 3y = 56. \\ 8x - 5y = 24. \end{cases}$

15. $\begin{cases} \frac{x}{2} + 8 = 5y - 6. \\ \frac{x}{4} + 12 = 6y - 9. \end{cases}$ **16.** $\begin{cases} \frac{2x + 3y}{5} = 2y - 4. \\ \frac{x}{3} - \frac{y}{4} = \frac{x + 2}{5}. \end{cases}$ **17.** $\begin{cases} \frac{3x - y}{3} = \frac{y}{4} + 6. \\ \frac{x}{2} + \frac{2y}{8} = \frac{3x}{4} + 1. \end{cases}$

18. $\begin{cases} \frac{2x - 3y}{6} = \frac{x}{3} + 1. \\ \frac{x}{3} + \frac{5x}{2} = x + 5. \end{cases}$ **19.** $\begin{cases} \frac{3x + 2y}{8} = \frac{2y}{3} + 5. \\ \frac{x}{4} - \frac{3y}{9} = \frac{y}{12}. \end{cases}$ **20.** $\begin{cases} \frac{4x - 40}{8} = \frac{y}{4\frac{4}{5}}. \\ \frac{4y - 60}{12} = \frac{x}{3} + 1. \end{cases}$

21. $\begin{cases} \frac{x}{2} + \frac{x}{3} = \frac{y}{4}. \\ \frac{z}{2} - \frac{y}{5} = \frac{x}{4}. \\ \frac{3x}{2} - \frac{2z}{3} = 4. \end{cases}$ **22.** $\begin{cases} \frac{2x}{8} + \frac{6y}{4} = 6. \\ 6y - 2 = z. \\ \frac{3y}{2} + \frac{x}{6} = 5. \end{cases}$ **23.** $\begin{cases} 3x - y + z = 13. \\ 2y - 3x - 2z = 1. \\ \frac{x}{3} + \frac{y}{2} = 2z + 1. \end{cases}$

1. If in the equation $x+2y=10$, the value of y is 4, what is the value of x?

2. If in the equation $2x+y=15$, the value of x is $y+1$, what is the value of y?

3. Substitute the value of x ($x=y+2$) in the following equation, and find the value of y: $x+3y+2x=42$.

4. If $x=2y+1$, what is the value of y in the following equation: $4x+3y=26$?

5. Find the value of x and of y from the two equations $x=y+2$; $3x-2y=12$.

Find by substitution the values of x and y in:

6. $\begin{cases} x+1=y. \\ 3x-2y=7. \end{cases}$ 7. $\begin{cases} x+y=11. \\ 2x-y=4. \end{cases}$ 8. $\begin{cases} x+2y=14. \\ 3x-2y=18. \end{cases}$

9. $\begin{cases} 2x+3y=6. \\ x+4y=14. \end{cases}$ 10. $\begin{cases} 2x+4y=30. \\ 3x-2y=21. \end{cases}$ 11. $\begin{cases} 2x+2y=40. \\ 3x-y=12. \end{cases}$

12. $\begin{cases} 3x+6y=33. \\ 4x-8y=12. \end{cases}$ 13. $\begin{cases} 2x+3y=54. \\ 4x-2y=28. \end{cases}$ 14. $\begin{cases} 2x-y=12. \\ 5x-4y=3. \end{cases}$

15. $\begin{cases} \frac{x}{3}+\frac{x}{5}=x-16. \\ \frac{3y-10}{5}=\frac{x}{3}. \end{cases}$ 16. $\begin{cases} 3x+5y=x-2\frac{1}{2}. \\ \frac{x}{3}-\frac{y}{2}=\frac{x-1}{4}+1. \end{cases}$ 17. $\begin{cases} \frac{9x}{6}+y=25. \\ \frac{7x-2y}{5}=\frac{x}{3}. \end{cases}$

18. $\begin{cases} \frac{2x+y+4}{8}=\frac{y}{2}. \\ \frac{x}{7}+\frac{3x}{10}=\frac{x-8}{2}. \end{cases}$ 19. $\begin{cases} \frac{x}{y}-8=6. \\ \frac{x+y}{9}=5. \end{cases}$ 20. $\begin{cases} \frac{2x+y+3}{3}=13\frac{1}{3}. \\ \frac{y-x-9}{4}=2\frac{1}{2}. \end{cases}$

21. Given $2u-x+3y-z=7$; $3u+2x-2y+3z=53$; $4z-3x=12$; $4y-\frac{u}{2}=13$, to find the values of u, x, y, and z.

22. Given $\frac{u+6}{8}+y-\frac{z}{2}=12$; $3x+2z-u-\frac{y}{4}=15$; $4z+u-3y=6$; $\frac{2y}{6}+\frac{x}{2}-\frac{u}{6}=5$, to find the values of u, x, y, and z.

Find (in either of three ways) the value of x and of y in :

1. $\begin{cases} 2x+2y=26. \\ x-y=3. \end{cases}$

2. $\begin{cases} 3x+y=28. \\ 2x-y=2. \end{cases}$

3. $\begin{cases} 2x-y=4. \\ 3y+2x=84. \end{cases}$

4. $\begin{cases} 2x-2y=2. \\ 3x+y=35. \end{cases}$

5. $\begin{cases} 2x+10y=44. \\ x-3y=6. \end{cases}$

6. $\begin{cases} 5x-y=4. \\ 2x+2y=40. \end{cases}$

7. $\begin{cases} 3x-4y=0. \\ 2x+y=55. \end{cases}$

8. $\begin{cases} \frac{x}{3}+y=13. \\ x+y=25. \end{cases}$

9. $\begin{cases} \frac{x}{4}+y=16. \\ 2x+\frac{y}{2}=22. \end{cases}$

10. $\begin{cases} \frac{x}{2}-\frac{y}{4}=18. \\ \frac{x}{3}+\frac{y}{4}=6. \end{cases}$

11. $\begin{cases} 2x-2y=6. \\ x-\frac{y}{3}=23. \end{cases}$

12. $\begin{cases} \frac{x}{2}-\frac{y}{2}=1. \\ \frac{x}{3}+\frac{y}{2}=9. \end{cases}$

13. $\begin{cases} \frac{3x}{4}-\frac{2y}{3}=3. \\ \frac{2x}{8}+\frac{3y}{2}=32. \end{cases}$

14. $\begin{cases} 4x-\frac{3y}{4}=52. \\ \frac{5x}{3}+\frac{y}{4}=14. \end{cases}$

15. $\begin{cases} 2x+\frac{4y}{6}=10. \\ \frac{x}{9}-\frac{y}{3}=-5. \end{cases}$

16. $\begin{cases} \frac{x+y}{6}=5. \\ \frac{2x+3y+10}{8}=10. \end{cases}$

17. $\begin{cases} \frac{3x}{4}-4=y-1. \\ \frac{2x+3y}{7}=12. \end{cases}$

18. $\begin{cases} \frac{2x}{5}+\frac{4y}{12}=20. \\ \frac{5y}{x}=4. \end{cases}$

19. $\begin{cases} \frac{6x-8}{4}=y-2. \\ \frac{3y+12}{6}=\frac{2y}{3}. \end{cases}$

20. $\begin{cases} \frac{4x+8y}{16}=\frac{2x-16}{2}. \\ \frac{3x+12}{3}=4y+8. \end{cases}$

21. $\begin{cases} \frac{6x}{3}+\frac{2y}{8}=19. \\ \frac{4x}{9}-3=\frac{y}{4}. \end{cases}$

22. $\begin{cases} \frac{2x+3y}{3}=x+9. \\ \frac{x}{3}+\frac{2x}{9}+\frac{y}{4}=y. \end{cases}$

23. $\begin{cases} \frac{x}{4}+\frac{4y}{3}=\frac{3x}{2}+1. \\ \frac{x}{3}+\frac{y}{6}=10\frac{2}{3}. \end{cases}$

24. $\begin{cases} \frac{9x+7y}{8}=2y-9. \\ \frac{5x-y}{4}=\frac{2y-10}{3}. \end{cases}$

25. $\begin{cases} 2x+\frac{y}{2}+\frac{3z}{4}=28. \\ \frac{4x}{3}-\frac{y}{4}+z=22. \\ x+\frac{3y}{8}+\frac{3z}{2}=33. \end{cases}$

26. $\begin{cases} \frac{2x}{5}+\frac{3y}{10}-\frac{z}{5}=9. \\ \frac{z}{x}+8=10. \\ \frac{4x+3y-2z}{3}=30. \end{cases}$

Miscellaneous.

1. Remove the parenthesis from $a+(b-c)$; $(a-b)-c$; $a-(b+c)$; $a-(b-c)$; $a+b-(c+d-e)$.

2. $(a+b)\times(a+b)=?$ $(a+b)\times(a-b)=?$

3. $(2a-3b+4c)\times(3a+2b-5c)=?$

4. $(5c+3)\times(c+d)-5\times(4c-2d)+12c=?$

5. $(ac-bc+ad-bd)\div(a-b)$; $(a^2-b^2)\div a+b)$.

6. $(a^4-b^4)\div(a^2-b^2)$; $(a^5+b^5)\div(a+b)$.

7. $a+b-[c-d-(g+h-i)+4g-i]=?$

8. Separate into two factors: $ab-b$; x^2-x; $ax-2ay+3az$.

9. $\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}=?$ $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}=?$

10. a and b are two numbers. Express (a) the square of the sum of both numbers; (b) the sum of the squares of both numbers increased by double their product; (c) the square of the difference of both numbers; (d) the difference of the squares of the two numbers; (e) the product of the sum and the difference of both numbers; (f) the cube of the sum of both numbers.

11. Find the value of x in: $ax=m$; $3x-12x=-36$; $12x-nx=a$; $mx+nx=a$; $ax+x=m$.

12. Find the value of x: $3x-10=14-x$; $ax-b=c$; $12x-7+4x=2-4x+8x$; $8x+30-6x=12x-6-16x$; $ab-ax=c$; $ax=c-bx$; $ax-c=bx-d$.

13. Find the value of x: $13\frac{3}{4}-\dfrac{x}{2}=2x-8\frac{3}{4}$; $2x+7+\dfrac{3x}{2}=6x-23$; $77-\dfrac{48}{x}=53$; $\dfrac{ab}{x}=c$.

14. Find the value of x: $\dfrac{a}{x}=b+c$; $ax=\dfrac{b}{c}$; $ab-\dfrac{x}{a}=c$; $ab-\dfrac{a}{x}=bc$; $\dfrac{x}{2}+\dfrac{x}{3}+\dfrac{x}{4}=7x-712+\dfrac{x}{5}$.

15. Find x in the following proportions: $4:5=18:x$; $x:18=9:31\frac{1}{2}$; $x:\dfrac{a}{c}=\dfrac{c}{9}:\dfrac{a}{b}$; $\dfrac{a}{b}:x=\dfrac{c}{d}:\dfrac{b}{d}$; $6:x=x:25$; $8:x=x:20$; $\frac{1}{2}:\dfrac{x}{4}=3:8$; $ax:2ab=b:c$; $2ac:bx=3a:4c$.

1. Find two numbers whose sum is 14 and whose difference is 2.

2. Find the numbers whose sum is 26, and half of whose difference is 5.

3. Find two numbers such that one shall be as much greater than 14 as the other is less than 14, and $\frac{1}{3}$ of their difference is equal to 4.

4. If $\frac{1}{2}$ of John's money is equal to $\frac{1}{3}$ of James's, and $\frac{1}{2}$ of James's money is 5 more than $\frac{1}{3}$ or John's money, how much has each?

5. If 6 apples and 3 oranges sell for 21 cents, and 4 apples and 8 oranges sell for 48 cents, what is the price of each?

6. Find two numbers such that 20% of one is equal to 50% of the other, and 75% of their difference is equal to 3.

7. The difference between two numbers is 10. Three times the larger number added to twice the smaller number is 105. Numbers?

8. A man is 10 years older than his wife, and she is 30 years older than her daughter. The sum of the ages of all three is 100 years. What is the age of each?

9. A boy is 10 years old, and his father is 40 years old. In how many years will the father's age be double that of his son?

10. James is $\frac{1}{2}$ older than John, but in 6 years he will be only $\frac{1}{3}$ older. How old is each?

11. If 8 lb. of coffee are worth 3 lb. of tea, and 7 lb. of tea are worth 90 cents more than 15 lb. of coffee, what is the price of each per pound?

12. A merchant gains each year $\frac{1}{3}$ of his capital, and at the end of each year withdraws \$1000. At the beginning of the fourth year his capital was double what it was three years before. What was his capital originally?

13. A and B have together \$40. If A should give to B \$4, then A would have \$10 more than B. How much has each?

14. A man having 5 children gave to the first half of all the apples he had, less 8 apples; to the second, half of what remained, less 8 apples; and in the same way to the other three children, giving the last child 20 apples. How many apples had he in the first place?

1. A man gives away $\frac{1}{3}$ of his money and \$50, and then has \$60 more than he gave away. How many dollars had he at first?

2. A man being asked how much he sold a horse for, answered, "By the trade I lost 10%; but if I had gained 10% I should have received \$120 more than I did receive." What did he sell the horse for?

3. A class of 80 pupils is divided into two sections; $\frac{1}{8}$ of the A section exceeds $\frac{1}{10}$ of the B section by 7. How many pupils in each section?

4. A clerk worked for a merchant with the understanding that he was to receive \$2 a day for every day he worked, and that he was to pay the merchant \$1.25 every day he was absent. At the end of 50 days he received \$54.50. How many days did he work?

5. A cistern has two pipes entering into it. One pipe can fill it in 4 hours and the other in 6 hours. How long will it take them both to fill it?

6. In an evening company there were as many ladies as gentlemen present. When 4 ladies left, the number of ladies were to the number of gentlemen as 4 : 5. How many persons were present?

7. A said to B, "Give me \$1 of your money, and I shall have twice as much as you." B answered, "Give me \$1 of your money, and I shall have exactly as much as you." How much had each?

8. 20 pigeons and 14 chickens cost \$15. 10 pigeons and 10 chickens cost \$9. What does each cost?

9. If a sheep costs \$7 and a calf costs \$5, how many sheep and calves can be bought for \$170?

10. If by spending \$20 a week a man's money will last 4 weeks longer than it would by spending \$25 a week, how much money had he?

11. A gentleman divided \$1000 among his four daughters so that \$150 added to Jane's part was equal to Sarah's part, twice Ellen's part was \$50 more than Jane's part, three times Mary's part was \$200 more than Sarah's part, and half of Mary's part was equal to \$50 less than Ellen's part. What was each daughter's share?

12. *Find two numbers* whose sum is 23 and whose difference is 7.

1. Three boys bought fruit at the same price. James bought 4 apples, 2 pears, and an orange for 14 cents; John bought 2 oranges, 3 pears, and 2 apples for 15 cents; William bought 5 pears, 3 oranges, and an apple for 19 cents. What was the price of each?

2. A man puts one-third of his money at interest at 5%, and the remainder at $6\frac{1}{4}\%$. If he receives \$2100 interest yearly, what is his principal?

3. 20 cows eat a certain quantity of hay in 40 days. 240 sheep and 20 cows will eat the same quantity in 10 days. How long would the same quantity last 240 sheep?

4. Find two numbers whose difference and quotient is 5.

5. Find two numbers whose sum is 184 and whose ratio is that of 6 to 8.

6. A certain sum of money at simple interest amounts to \$1545 in 6 mo., and \$1567.50 in 9 mo. What is the principal and rate?

7. Find two numbers in the ratio of 2 to 3 such that if 4 be taken from each the numbers will be in the ratio of 1 to 2.

8. A man bought some oranges at the rate of 2 for 3 cents, and some apples at the rate of 4 for 5 cents. He sold them all for 2 cents apiece, thereby gaining 60 cents. How many of each kind did he buy?

9. A and B can perform a certain piece of work together in 6 days, A and C can perform it in $4\frac{1}{2}$ days, and B and C in $5\frac{1}{2}$ days. In how many days could each perform the work?

10. A man bought \$5000 worth of railroad 4's and 5's at par. His yearly income from his 4 per cent bonds is \$65 more than what he receives in interest from the 5 per cent bonds. What sum has he invested in each kind?

11. If the numerator of a certain fraction is increased by 8 and the denominator is decreased by 8, the value of the fraction will be 2. If the numerator is decreased by 3 and the denominator is increased by 3, the value of the fraction is $\frac{1}{6}$. What is the fraction?

12. A number of 2 digits is 4 times the sum of its digits; the tens digit is 4 smaller than the units digit. What is the number?

1. If the Nile is $\frac{6}{8}$ as long as the Amazon, and $\frac{1}{2}$ the length of the Amazon is 200 miles less than $\frac{2}{3}$ the length of the Nile, and $\frac{1}{4}$ the length of the Danube is equal to $\frac{1}{3}$ the length of the Amazon, what is the length of each?

2. St. Peter's Church in Rome is 227 ft. higher than Bunker Hill Monument, which is 334 ft. lower than the Washington Monument. If Bunker Hill Monument is 6 ft. less than one-half the height of St. Peter's, what is the height of each?

3. At what time between 3 and 4 o'clock will the hands of a clock be together? At what time between 6 and 7?

4. How shall $3000 be divided so that the ratio of one amount to the other will be as 7 to 8?

5. A tank can be filled by two pipes in $5\frac{1}{2}$ hours, and by one alone in 9 hours. How long would it take the other pipe to fill it?

6. A and B can do a piece of work in $3\frac{3}{7}$ days. If A can do it in 6 days, how long would it take B?

7. What two numbers are there whose difference and whose quotient is 5?

8. How many gallons of molasses at a cents a gallon must be mixed with m gallons at b cents a gallon, that the mixture may be worth c cents a gallons?

9. Divide the number a into two such parts so that the first is to the second as $l : m$.

10. Divide a dollars into three such parts that the first is to the second as $l : m$, and the second to the third as $n : o$.

11. What is the amount of a dollars at compound interest for m years at n per cent?

12. Find two numbers which have a ratio of a to b, but if m be added to each the result will be in the ratio of c to d.

13. If a stack of hay will last a horse a months and a cow b months, how long would it last m horses and n cows?

14. Find two numbers whose difference, sum, and product are in the proportion of 1 : 4 : 30.

15. At what time between 12 and 1 o'clock are the minute-hand *and the hour-hand* of a clock in a straight line?

SECTION III.

INVOLUTION AND EVOLUTION.

1. Use each of the following numbers twice as a factor, and give the products: 3; 4; 5; 8; 10; 12.

2. Use each of the numbers given in exercise 1 three times as a factor, and give the products.

3. What is the fourth power of 2? of 3? of 4? of 1?

4. What is the third power of 4? of 5? of 6? of 7?

5. Name the squares of the numbers from 1 to 10 inclusive.

6. Name the squares of the numbers from 11 to 20 inclusive.

7. Name the cubes of the numbers from 1 to 6 inclusive.

8. Name the cubes of the numbers from 7 to 12 inclusive.

9. What is the square of $\frac{1}{2}$? of $\frac{1}{4}$? of $\frac{2}{3}$? of .5?

10. What is the cube of $\frac{1}{2}$? of $\frac{2}{3}$? of $\frac{3}{4}$? of .3?

11. Find the indicated powers of the following numbers: 18^2; 20^3; $(1\frac{1}{3})^2$; $.8^2$; 2.5^3.

Find the value of:

12. $8^2 \times 3$.
13. $3^3 \times 4$.
14. $9^2 \div 3^2$.
15. $8^3 \div 4^2$.
16. $(\frac{3}{4})^2 \div \frac{1}{4}$.
17. $3^2 \times (\frac{1}{2})^3$.
18. $(4\frac{1}{2})^3 \times .2^3$.
19. $.05^3 \times (\frac{1}{2})^4$.
20. $3^3 \div 3^2 \times (\frac{1}{2})^2$.
21. $(\frac{1}{2})^3 \times 8^2 + 10^2$.
22. $9^2 \times (\frac{1}{3}) + 16^2$.
23. $(\frac{2}{3})^2 + (\frac{1}{2} + \frac{1}{4})^2$.
24. 1.5^2.
25. $(.6\frac{1}{4})^3$.
26. $(.03\frac{1}{3})^4$.
27. $(.16\frac{2}{3})^3$.

28. Raise to the second power: 40; 84; 99; 100; 560.

29. Raise to the second power: 830; 326; 689; 999; 1000.

Find the value of:

30. 286^2.
31. 4.58^2.
32. $.0075^2$.
33. 3.01^3.
34. 75^4.
35. 1.0501^2
36. 80.2^4.
37. $(4\frac{5}{8})^3$.
38. 8^5.
39. 4.45^3.
40. $(8\frac{3}{4})^4$.
41. $(72\frac{1}{2})^3$.

1. How many places are there in the square root of an integer expressed by one figure? by two figures? by three figures? by four figures?

2. What is one of the two equal factors of 81? of 121? of 256? of .01? of .0016? of 1.44?

3. What is the square root of 196? of 289? of 361? of 324? of 2.89? of 3.61?

4. What is the square of the smallest number of two figures? of the largest number of two figures? of the smallest number of three figures? of the largest number of three figures?

5. Can you tell from answers of the last questions how the number of figures in the square root of a given power may be determined?

6. How many figures in the square root of 625? of 6400? of 2116? of 9801? of 998,001? of 1,102,500?

7. What orders of numbers in the square root of 576? of 3969?

8. What is the greatest square that can be found in 120? in 300? in 400? in 270?

9. Extract the square root of 3600; of 6400; of 8100.

10. If the square root consists of tens + units, what must the power consist of? From this answer can you derive the expression $\text{tens}^2 + (2 \text{ tens} + \text{units}) \times \text{units}$?

11. By the aid of the formula, extract the square root of 576. The root of this power must consist of how many figures? Show how you can find the square of tens + units. The greatest square of tens found in 576 is what? Subtracting and what remains? This is equal to what? How can we find the units figure? What may be the trial divisor? The units may be what? What is the complete divisor? By multiplying as indicated, what do we find? What, then, is the square root of 576?

$$
\begin{array}{rl|l}
 & & t+u \\
t^2+(2t+u)\times u=576 & & 2 \quad 4 \\
t^2=400 & & \\
\hline
2t=40 & 176=(2t+u)\times u & \\
u=\ 4 & & \\
\hline
2t+u=44 & 176=(2t+u)\times u &
\end{array}
$$

In the same way extract the square root of :

1. 1024.	**5.** 5329.	**9.** 3249.	**13.** 6889.
2. 1849.	**6.** 1225.	**10.** 5776.	**14.** 8836.
3. 2601.	**7.** 2116.	**11.** 4489.	**15.** 7921.
4. 3844.	**8.** 1296.	**12.** 5476..	**16.** 9409.

17. The square of tenths gives what? of eighths? of halves? of hundredths?

18. Extract the square root of $\frac{9}{49}$; of $\frac{64}{81}$; of .09; of .36; of .0025.

19. Extract the square root of $\frac{121}{169}$; of $\frac{289}{361}$; of .0169; of 9.61.

By the formula, extract the square root of :

20. 17.64.	**23.** 12.25.	**26.** 33.64.	**29.** 92.16.
21. 39.69.	**24.** 31.36.	**27.** 47.61.	**30.** 98.01.
22. 29.16.	**25.** 22.09.	**28.** 77.44.	**31.** 62.41.

32. In finding the square root of 15,129, we know that there are how many figures in the root? Regarding hundreds and tens as tens and units of tens, find the first two figures. Regarding these two figures as tens, find the units figure.

Extract the square root of :

33. 53,361.	**38.** 273,529.	**43.** 355,216.	**48.** 53,870.41.
34. 116,964.	**39.** 225,625.	**44.** 459,684.	**49.** 125,741.16.
35. 64,516.	**40.** 77,841.	**45.** 938,961.	**50.** 216,969.64.
36. 132,496.	**41.** 401,956.	**46.** 646,416.	**51.** 2576.5776.
37. 81,796.	**42.** 238,144.	**47.** 502,681.	**52.** 3622.8361.

53. Reduce 2 to ten thousandths, and extract the square root.

Extract the square root of the following numbers to two places of decimals :

54. 40.	**58.** 3.5.	**62.** 160.4.	**66.** 5.006.	**70.** $\frac{5}{12}$.
55. 242.	**59.** 16.2.	**63.** 32.728.	**67.** 13.048.	**71.** $\frac{16}{21}$.
56. 1380.	**60.** 7.25.	**64.** 4.837.	**68.** 117.45.	**72.** $\frac{8}{15}$.
57. 1675.	**61.** 8.08.	**65.** 900.6.	**69.** 27.3.	**73.** $\frac{2}{3}$.

1. A square lot containing 5476 sq. rd. of land is how long?

2. How long is a square field containing 34,596 sq. ft.? containing 1 A. 31,516 sq. ft.? containing 6 A.?

3. How long and wide is a rectangular piece of land containing $\frac{1}{2}$ of an acre, whose length is twice its breadth?

4. How far must a piece of land 8 rods square be extended to contain 2 acres.

5. A rectangle is 468 ft. long, 138 ft. wide. Find the side of a square which has the same area.

6. How many more feet of fence will it take for a rectangular lot 42 rd. long, 28 ft. wide than for a square lot of the same size?

7. If it takes 21,904 blocks 8 inches square to pave a square courtyard, how many blocks on each side, and how large is the courtyard?

8. How many paving-stones 9 inches square will it take to pave a court containing 53,824 sq. ft.?

9. The sides of three squares are 6 in., 8 in., and 10 in. respectively. What is the length of one side of a square that is equal in area to the three squares?

10. Find the dimensions of a floor which contains 384 sq. ft., and whose length and width are as 3 to 2.

11. I have a half-acre lot of land whose width is $\frac{3}{4}$ of its length. How far from centre to centre must the posts of a fence be set that will give panels of equal length as nearly as possible to 12 ft. long?

12. How many rods of fence will be required to inclose 6 acres of land, if the ratio of width to length is as 1 to 4?

13. The difference of the squares of two numbers is 136. If each number is increased by 2, the difference of their squares will be 152. What are the numbers?

14. What will it cost to fence a 10-acre field which is in the form of a square, panels 10 ft. long, 4 rails to the panel, posts costing \$15 a hundred, rails \$5 a hundred, two men working 14 days each at \$1.75 a day?

15. A cistern 6 ft. deep holds 1240 gallons. What are the other *dimensions if the* width is equal to the length?

1. How many figures has the third power or cube of the smallest number of one figure? of the largest number of one figure? of the smallest number of two figures? of the largest number of two figures? of the smallest number of three figures? of the largest number of three figures? From these facts deduce a rule for finding the number of figures in the cube root of a number.

2. How many figures in the cube root of 13,824? of 1,404,928? of 941,192?

3. Name the cubes of numbers from 1 to 12 inclusive.

4. What is the greatest cube that can be found in 30? in 100? in 500? in 1000? in 1500? in 1800?

5. If the cube root of a number consists of tens and units $(t+u)$, what must the power be? What is $(t+u)^3$? Can you reduce this quantity to the equivalent expression $t^3+(3t^2+3t\times u+u^2)\times u$?

6. By the aid of the formula, extract the cube root of 12,167.

$$
\begin{array}{rl|l}
 & & t+u \\
t^3+(3t^2+3t\times u+u^2)\times u = & 12{,}167 & 2 \quad 3 \\
t^3 = & 8{,}000 & \\
\hline
3t^2 = & 1{,}200 & 4{,}167 = (3t^2+3t\times u+u^2)\times u \\
3t\times u = & 180 & \\
u^2 = & 9 & \\
\hline
3t^2+3t\times u+u^2 = & 1{,}389 & 4{,}167 = (3t^2+3t\times u+u^2)\times u
\end{array}
$$

The cube of tens gives thousands. The greatest cube that can be found in 12,000 is what? Subtracted from 12,167 is what? This remainder is equal to what? How may the units figure be approximately found? The complete divisor is what? What result after multiplying this divisor by the units? What remainder? What is the conclusion?

In the same way extract the cube root of:

7. 512,000.	**11.** 314,432.	**15.** 103.823.	**19.** 47,437,928.
8. 32,768.	**12.** 250,047.	**16.** 35.937.	**20.** 71,473,375.
9. 68,921.	**13.** 970,299.	**17.** 2048.383.	**21.** 105,154,048.
10. 91,125.	**14.** 912,673.	**18.** .704969.	**22.** 736,314,327.

1. Reduce 4 to millionths, and extract the cube root.

Extract the cube root of the following numbers to two places of decimals :

2. 60.	5. 4.8.	8. 112.05.	11. $\frac{1}{2}$.	14. .08.
3. 180.	6. 14.43.	9. 7.0008.	12. $\frac{3}{40}$.	15. .06$\frac{1}{3}$.
4. 1600.	7. 6.827.	10. 13.65.	13. $\frac{9}{20}$.	16. .4$\frac{1}{3}$.

17. What is the length of one side of a cubical block of wood that contains 2197 cu. in.? that contains 42,875 cu. in.?

18. What are the dimensions of a cubical cistern that contains 658,503 cu. in.? that contains 8 cu. yd.?

19. What is the inside measurement of a cubical box that contains a bushel of wheat? of a cubical bin that contains 50 bushels of wheat?

20. What is the inside measurement of a cubical cistern that contains 389,017 cu. in.? of a cubical cistern that contains 100 gallons of water?

21. What is the edge of a cube that contains as many cubic feet as a plank 18 ft. long, 9 in. wide, and 2$\frac{1}{2}$ in. thick?

22. There are 152$\frac{1}{8}$ sq. rd. in a lot of land whose width is to its length as 2 to 3. What are the dimensions of the lot?

23. What are the dimensions of a cubical pile of stone that contains 14 cords?

24. A box containing 27 cu. ft. is twice as wide as it is deep, and twice as long as it is wide. What are the inside dimensions of the box?

25. What is the length of a cubical bin that will contain as much wheat as a bin that is 18 ft. long, 4$\frac{1}{2}$ ft. wide, and 6$\frac{1}{2}$ ft. deep?

26. What is the length of a quarter-section of land? Make problems about cost of fencing, etc.

27. A cistern whose dimensions are in the proportion of 2 : 3 : 4 has a capacity of 9000 gallons. What are the dimensions?

28. The width of a rectangular field containing 3$\frac{1}{2}$ A. is $\frac{8}{9}$ of its *length. Required* the distance around it?

SECTION IV.

EXERCISES IN GEOMETRY AND MENSURATION.

1. Show what is meant by *space.*

2. Define *matter; body; solid* or *volume.*

3. Define *surface; line; point.*

4. How many dimensions has a volume? a surface? a line? a point?

5. Represent a straight line. Observe its direction and its distance between two points. Define *straight line.*

6. Represent a curved line. Observe its direction. Define *curved line.*

7. Define *vertical line; horizontal line; inclined line.* Give examples.

8. When are lines *parallel?* When is a line *perpendicular* to another? When is a line *oblique* to another? Give examples of each.

9. Draw free-hand as nearly as you can two horizontal lines that are parallel; two curved lines that are parallel; a line that is perpendicular to a vertical line; a line that is perpendicular to an inclined line; a line that is oblique to an inclined line.

10. Can you make, by means of rule and compasses, lines that are exactly parallel?

11. Can you divide a line into two equal parts?

12. Can you divide a line into three equal parts? any number of equal parts?

13. Name some units of length. Define unit of length.

14. What is the standard unit of length in this country? How determined? What is the standard unit of length in the metric system? How determined?

1. Repeat the table of long measure; surveyors' measure; measure of length, metric system. Give the origin and meaning of each term used.

2. Draw to a convenient scale lines representing 110 ft.; 330 yd.; 120 rd. To what scale are these lines drawn? Divide each line into six equal parts.

3. Draw two straight lines in the same plane extending in different directions. The difference in direction of these lines is an angle. Define *angle.* Define *vertex.*

4. Draw an angle formed by lines extending perpendicularly from each other. This is a right angle. Define *right angle.*

5. Draw an angle formed by lines inclined toward or from each other. This is an oblique angle. Define *oblique angle.*

6. Draw an oblique angle less than a right angle. This is an acute angle. Define *acute angle.*

7. Draw an oblique angle greater than a right angle. This is an obtuse angle. Define *obtuse angle.*

8. Point out below a right angle; an acute angle; an oblique angle; an obtuse angle.

a *b* *c* *d*

9. What is the unit of measuring angles? How obtained?

10. Make several angles and, by means of a protractor, measure their magnitude.

11. By means of a protractor lay off on a given line an angle of 90°; of 45°; of 60°; of 135°.

12. By means of a protractor make an angle equal to each of the above angles (Exercise 8).

13. By means of compasses make an angle equal to a given angle.

14. By means of compasses make an angle equal to double the size of a given angle; three times the size of a given angle; *one-half the size of* a given angle.

1. Draw two lines crossing each other. How many angles are formed by these lines? Point out the two angles whose sides extend in opposite directions. These are vertical angles. Define *vertical angles.*

2. Draw two angles that have a common vertex and a common side between them. These are adjacent angles. Define *adjacent angles.*

3. Measure with the protractor the sum of two adjacent angles.

4. Can you show without a protractor that the sum of two adjacent angles is equal to two right angles?

5. Find the supplement or adjacent angle of each of the following angles: 40°; 80°; 60°; 120°; 25°; 160°; 110°.

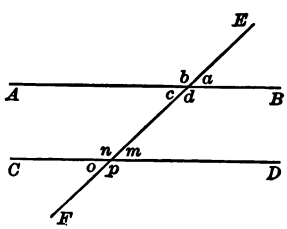
6. Compare with a protractor the size of any two vertical angles.

7. Can you show without a protractor that two vertical angles are always equal to each other?

8. Draw two lines crossing each other. Point out the adjacent angles; vertical angles. Find the sum of all the angles.

9. If one of the angles formed by two intersecting lines is 90°, what are the others? If one of the angles is 20°, what are the others?

10. In this figure, *AB* is parallel to *CD.* Point out the *internal angles; external angles; exterior-interior angles; alternate-interior angles.* Show with and without a protractor what angles are equal. What angles are together equal to two right angles?



11. Put the edge of a rule upon the top of your desk or upon the blackboard. If the rule touches the surface at every point, it is a plane surface. Define *plane surface.* Draw on a plane surface a figure bounded by straight lines. This is a polygon. Define *polygon.*

12. Draw, name characteristics, and define each of the following named figures: *triangle; equilateral triangle; isosceles triangle; scalene triangle; right-angled triangle; oblique-angled triangle; acute-angled triangle; obtuse-angled triangle;* equiangular triangle.

1. Define and give illustration of each of the following parts of a triangle: *base; sides; altitude; vertex.*

2. Show that the sum of two sides of a triangle is always greater than the third side.

3. Show in any way you can that the sum of the angles of a triangle is equal to two right angles.

4. Draw a triangle. Prolong one side. Show that the exterior angle is equal to the sum of the two opposite interior angles.

5. Show that the angles of an equilateral triangle are equal.

6. Show that the angles opposite the equal sides of an isosceles triangle are equal.

7. Show that two triangles are equal if two sides and included angle of one triangle are equal to the two sides and included angle of the other.

8. Show that two triangles are equal if a side and two angles of one triangle are equal to a side and two angles of the other.

9. Show that two triangles are equal if three sides of one triangle are equal to three sides of the other.

10. Draw two equal right-angled triangles; two equal obtuse-angled triangles; two equal acute-angled triangles.

11. Draw a triangle having a base two inches long, one angle of 90°, and one angle of 45°. What is the other angle? What is the length of each of the other sides?

12. Draw a triangle whose sides are as follows:

What is the size of each angle?

13. Draw a right-angled triangle whose sides are 3 inches, 4 inches, and 5 inches.

14. Draw to scale an isosceles triangle whose sides represent a length of 42 feet, and whose vertex is an angle of 25°. How long is the base?

15. Draw a triangle whose sides are 4 inches, 5 inches, and 6 *inches long.* Size of the angles? What kind of a triangle is it?

1. Given a line 4 inches long; two angles, 64°, 102°. Construct a triangle. What kind of a triangle is it? Length of other sides?

2. Draw an equilateral triangle whose perimeter is 14 inches.

3. Draw an isosceles triangle whose base is 20 cm and the sum of whose sides is 60 cm. What are the angles?

4. Draw an isosceles triangle. Draw a line from the middle point of base to the vertex. What can you prove about the two triangles? about the angles at the vertex? about the base and the line dividing the triangle?

5. Draw an angle. Can you bisect it without the aid of a protractor?

6. Draw a straight line. Can you bisect it without measuring?

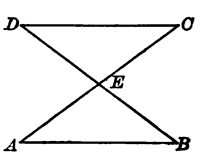
7. Can you let fall a perpendicular from a given point to a given line? Can you erect a perpendicular to a given line?

8. Draw a triangle. Bisect each of the three angles.

9. Divide an isosceles triangle into two equal triangles.

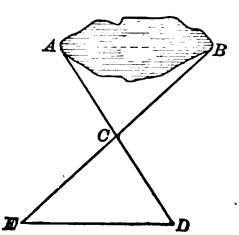
10. Draw a triangle whose two angles and included side are equal to the two angles and included side of another triangle. How do these triangles compare?

11. To prove that the triangle *DEC* is equal to the triangle *ABE*, what three parts of one triangle must be shown to be equal to the corresponding parts of the other triangle? If it is desired to know the length of the line *DC*, what lines must be measured?



12. Two sides of a triangular grass-plot are 48 feet and 36 feet. The angle included between these two sides is 70°. Draw to convenient scale. How long is the third side?

13. Find the length of a pond between the two points *A* and *B*. Fix the point *C*. Draw *AC* and *BC*. The lines *AC* and *BC* are how long? Prolong these lines how far? Why are the two triangles equal? How long is *ED*? How long is *AB*? Why?



1. Draw a quadrilateral. Draw diagonals. (How many?) Define *quadrilateral.* Define *diagonal.*

2. Draw and define *trapezium; trapezoid; parallelogram; rectangle; square; rhomboid; rhombus.*

3. Show with a protractor that the sum of the angles of a quadrilateral is equal to 360°. Can you show this without the aid of a protractor?

4. Can you show that the diagonal of a parallelogram divides the parallelogram into two equal parts?

5. Can you show that any two consecutive angles of a parallelogram are equal to two right angles?

6. Can you show that the opposite sides of a parallelogram are equal?

7. Can you show that the opposite angles of a parallelogram are equal?

8. Can you show that the diagonals of a parallelogram bisect each other?

9. If one angle of a parallelogram is 60°, what are the other angles?

10. Construct a rhombus with one side 2 inches long and with one angle of 40°. What are the other sides and angles?

11. Draw two lines each one inch long, and two lines each two inches long. Draw an angle less than 90°. From these construct a parallelogram.

12. Draw four lines of unequal lengths. Draw an angle of 70°. From these construct a trapezoid. What is the altitude of this trapezoid? the length of median line?

13. Draw an isosceles trapezoid. What can you show of the angles adjacent to one of the parallel lines?

14. Can you divide a parallelogram into 2 equal parallelograms?

15. Can you divide a rhombus into 4 equal right-angled triangles?

16. Can you divide an isosceles trapezoid into a parallelogram and an isosceles triangle?

17. Draw and define a *pentagon; hexagon; heptagon; octagon; nonagon; decagon; dodecagon.*

1. Draw a polygon with a reëntrant angle; with two reëntrant angles.

2. Draw a polygon having equal sides and equal angles. Such a polygon is a regular polygon. Define *regular polygon.*

3. Into how many triangles may a polygon be divided. Make a general statement.

4. Find the sum of the interior angles of a pentagon; of an octagon; of any polygon. Make a general statement.

5. Bisect the angles of a regular polygon. What do you find in regard to the bisecting lines? Can you prove that this point is equally distant from all the corners and from all the sides?

6. Bisect the angles of an equilateral triangle; of a square; of a regular pentagon; of a regular hexagon. Find the angle at the centre in each case.

7. Can you construct, with the aid of a protractor, an equilateral triangle? a square? a regular pentagon? a regular hexagon? a regular octagon?

8. Can you make a hexagon having five right angles? having four right angles?

9. Can you divide a hexagon into three rectangles and four right-angled triangles? into a square and four isosceles triangles?

10. Draw an equilateral triangle. Can you construct another equal to it?

11. Draw a heptagon. Can you construct another equal to it?

12. Name several units of area. Define a unit of area.

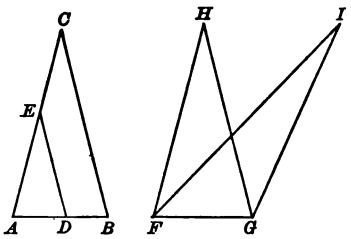
13. Show by diagram what is meant by 4 square feet; 8 square rods; 6^{qm}.

14. Repeat all the tables of square measure.

15. From what you have learned, give the formula for finding the area of a parallelogram, letting S = area or surface, a = base, h = altitude.

16. Give the formula for finding the base of a parallelogram when the area and altitude are given; for finding the altitude when the area and base are given. Give the formula for finding one side of *a square* when the area is given.

1. Give the formula for finding the area of a triangle when the base and altitude are given; for finding the base; for finding the altitude.



2. From what you have learned, show what three triangles are equal; what two triangles are equivalent; what two triangles are similar.

3. Obtain the formula for finding the area of a trapezoid, letting a and c represent the two parallel sides, and h the altitude.

4. Show how the area of any polygon is obtained. What measurements must be given?

5. Can you divide an irregular hexagon into right triangles and trapezoids?

6. Draw a regular pentagon. Draw a line from each angle to the centre. Let fall a perpendicular from the centre to each side of pentagon. Calling this perpendicular line the *apothem* of the pentagon, obtain the formula for finding the area of a regular pentagon (represent the perimeter by p and the apothem by r). In the same way obtain the formula for finding the area of any regular polygon.

7. Can you transform a parallelogram into an equivalent triangle?

8. Can you transform a triangle into an equivalent parallelogram?

9. Can you transform a trapezoid into an equivalent triangle? into an equivalent parallelogram?

10. Can you transform a regular pentagon into an equivalent triangle?

11. Can you divide a square into four equal squares? into six equal rectangles? into eight equal right-angled triangles?

12. Can you divide a rectangle into four equal rectangles?

13. Can you divide a triangle into three equivalent triangles? into any number of equivalent triangles?

14. Can you divide a parallelogram into six equal parallelograms? *into any number* of equal parallelograms?

1. How many acres in a square whose sides are 200 ft. long?

2. Find the area of a rectangle 18 yd. long, 32 ft. wide; 130 ft. long, 2 rd. wide; 315 ft. long, 80 ft. 3 in. wide.

3. What is the length of a rectangular piece of land which contains $\frac{3}{4}$ of an acre and which is 120 ft. wide?

4. Find the area of a triangle whose base is 20 ft. and whose altitude is 8 ft.; base 42 ft., altitude 96 ft.

5. What is the area of a parallelogram whose base is 42 ft. and whose altitude is 36 ft.? base 460 ft., altitude 134 ft.

6. The area of a certain field in the form of a parallelogram is 2 acres. If the length of one side is 400 ft., what is the perpendicular distance from that side to the opposite side?

7. A square plot of ground containing $\frac{1}{2}$ an acre is how long?

8. Find the area of a trapezoid whose parallel sides are 16 ft. and 18 ft., and whose altitude is 9 ft.

9. A board 8 in. wide at one end and 5 in. wide at the other must be how long to contain 2 square feet?

10. I have a lot of land in the form of a triangle whose base is 80 rd. and altitude 180 ft. Required the side of a square lot having the same area.

11. The area of a field in the form of a trapezoid is 4 A. 120 sq. rd. The parallel sides are 1680 ft. and 1240 ft. How far apart are these sides?

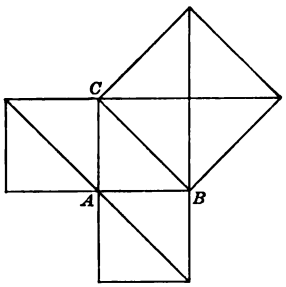
12. What is the area of a trapezium, the length of a diagonal being 120 ft., and the altitudes of triangles made by the diagonal being 28 ft. and 45 ft.?

13. From a lot of land in the form of a triangle having a base of 150 ft. and an altitude of 60 ft., a triangle is cut off, having a base of 60 ft., and an altitude of 18 ft. 9 in. What part of the original lot is cut off?

14. How many shingles will it take to cover a roof in the form of a trapezoid, the parallel sides being 40 ft. and 32 ft., and their distance apart 12 ft., the shingles being laid 4 in. to the weather?

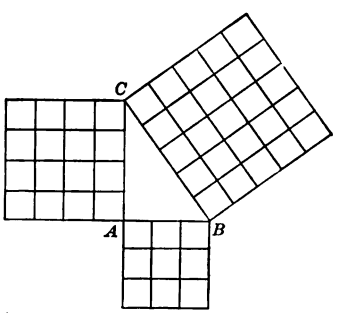
15. Draw to scale a plan representing a lot of land in the form of a regular pentagon, and find the area.

1. In the right-angled triangle ABC, the length of the base AB is equal to the length of the perpendicular AC. Draw the triangle ABC and erect a square upon each of the three sides. Show by measurements and by other proof, if you can, that a square erected upon the hypothenuse is equal to the sum of the squares erected upon the other two sides?



(How do the triangles in this figure compare in size?)

2. In the right-angled triangle ABC, the base is represented as 3 feet long and the perpendicular as 4 feet long. Can you show that the square of the hypothenuse is equal to the sum of the squares of the other two sides? Can you show this in every right-angled triangle whose base and perpendicular are in the ratio of 3 to 4?



3. Can you show that in every right-angled triangle the square of the hypothenuse is equal to the sum of the squares of the other two sides?

4. Give the formula for finding the hypothenuse when the base and perpendicular are given; for finding the perpendicular when the base and hypothenuse are given; for finding the base when the perpendicular and hypothenuse are given.

5. What is the length of the hypothenuse of a right-angled triangle whose base is 4 in. and whose perpendicular is 3 in.? whose base is 9 in. and whose perpendicular is 12 in.?

6. What is the base of a right-angled triangle whose perpendicular is 6 in. and whose hypothenuse is 10 in.? whose hypothenuse is 18 ft. and whose perpendicular is 12 ft.?

7. Find the perpendicular of a right-angled triangle whose base *is 15 yd. and whose* hypothenuse is 30 yd.

In a right-angled triangle, given :

1. The base 60 ft., perpendicular 80 ft., to find the hypothenuse.
2. The perpendicular 20 ft., base 15 ft., to find the hypothenuse.
3. The base 18 rd., perpendicular 40 rd., to find the hypothenuse.
4. The base 48 m, perpendicular 36 m, to find the hypothenuse.
5. The hypothenuse 80 ft., base 50 ft., to find the perpendicular.
6. The hypothenuse 150 rd., perpendicular 90 rd., to find the base.
7. The hypothenuse 28 yd., perpendicular 20 yd., to find the base.

8. Can you construct a square equal to the sum of two given squares ?

9. Can you construct a square equal to the difference between two given squares ?

10. Can you show that in an isosceles right-angled triangle a line extending from the right angle perpendicular to the hypothenuse is equal to half the base ?

11. The side of an equilateral triangle is 40 ft. Find the altitude; find the area.

12. The side of a square is 18 ft. Find the diagonal.

13. What is one side of a square if the diagonal is 100 ft. ?

14. Find the diagonal of the floor of a room 18 ft. long and 16 ft. wide; 26 ft. long and 20 ft. wide.

15. What is the length of a ladder which will reach to the top of a house 42 ft. high, if the foot of the ladder is placed 18 ft. from the house ?

16. How far from a house 28 ft. high must a ladder 38 ft. long be placed that the top of the ladder may reach to a point just 6 ft. from the top of the house ?

17. What is the diagonal on the floor of a room 26 ft. square ?

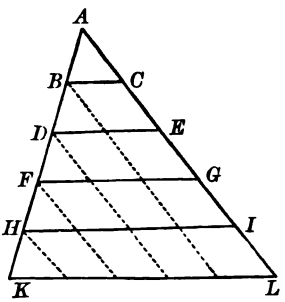
18. If A's house is 50 rods north of a given point, and B's house is 42 rods east of the same point, how far apart are the houses ?

19. The diagonal of a square lot is 36 sq. rd. What is one side ?

20. A room is 18 ft. long, 16 ft. wide, and 10 ft. high. What is the distance from a lower corner to the opposite upper corner ?

21. *Required the* diagonal of a square 6-acre field.

1. Point out in this figure the triangles that have the same shape. These triangles are *similar*. Can you prove that the angles of the triangle AFG are equal to the angles of the triangle AKL? The sides opposite equal angles are called *homologous* sides. Name the homologous sides of the two similar triangles AHI and AKL; of ADE and AFG. In what ratio is AD to AF? AE to AG? What proportion is there between the homologous sides of the two triangles ADE and AFG? between the homologous sides of AFG and AKL?



2. Draw a triangle. Draw a line through two sides of the triangle parallel to the third side. What can you say of the sides? Express the proportion. Show the proportion by applying numbers to sides.

3. Draw two similar triangles, each having the angles 45° and 90°; each having angles 60° and 40°.

4. Draw two similar equilateral triangles whose homologous sides are in the ratio of 3 to 4. Apply numbers to sides, and compute area of each. Compare the ratio of the areas with the ratio of the squares of the homologous sides.

5. Draw a pentagon. Draw other similar pentagons inside the first. Divide into triangles. Can you show that all similar pentagons are to each other as the squares of their homologous sides?

6. Can you show that all similar polygons are to each other as the squares of their homologous sides?

7. Can you show that the perimeters of similar polygons are to each other as any two homologous sides?

8. Draw a triangle. Draw a similar triangle whose sides are to those of the first as 3 to 1. Compare the areas of the two triangles.

9. Draw a triangle. Draw a similar triangle whose sides are one-fourth those of the first triangle. Compare their areas.

10. Draw a triangle. Draw a similar triangle equal in area to 9 *times the first* triangle. Compare corresponding sides.

1. Draw a regular hexagon. Draw another similar hexagon whose sides are three times as long as the sides of the first. How much larger is the second hexagon than the first?

2. Draw an octagon. Draw another similar octagon whose sides are one-half those of the first octagon. Compare their areas.

3. Draw a polygon. Draw another similar polygon four times as large as the first. Compare the corresponding sides.

4. Draw two similar pentagons, one having five times the perimeter of the other. What is the ratio of their sides? of their areas?

5. One side of a triangle containing 300 sq. ft. is 40 ft. The corresponding side of a similar triangle is 50 ft. What is its area?

6. If one side of a hexagon containing 4000 sq. ft. is 60 ft., what is the corresponding side of a similar hexagon whose area is 10,000 sq. ft.?

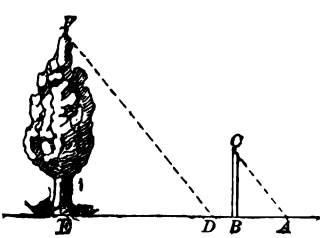
7. If the area of a piece of land in the form of an equilateral triangle is 840 sq. rd., what is the area of a similarly formed piece of land each of whose sides is $2\frac{1}{2}$ times as long?

8. If the base of a triangle is 12 ft. and its area is 20 sq. ft., what is the area of a similar triangle whose base is 40 ft.?

9. If one side of a triangular lot of land is 140 ft., what must be the length of the corresponding side of a lot of the same shape that is 3 times as large?

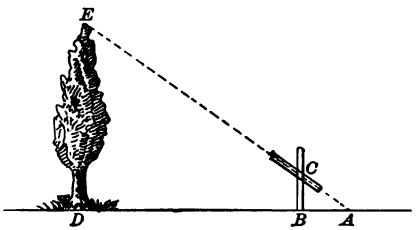
10. I have a triangular board 8 ft. long. At what distance from the base end shall I cut it to divide it into two equal parts?

11. To find the height of the tree EF: The shadow of the tree extends to the point $D = 26$ ft. from the foot of the tree. A pole 11 feet high casts a shadow 8 feet = the distance AB. What can you say of the two triangles ABC and DEF? $AB : DE = BC : ?$



12. If a post 6 ft. high casts a shadow $2\frac{1}{2}$ ft., how high is a house which at the same time casts a shadow 15 ft.? (Draw figure and *explain.*)

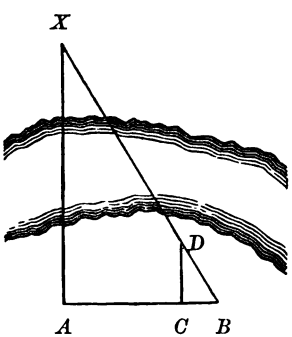
1. To find the height of the tree DE: A movable hollow tube or straight-edged beam is attached to the upright BC. How is the point A found? $BC = 6$ ft.; $AB = 8$ ft.; $BD = 36$ ft. What two similar triangles? What proportion can you make to find DE?



2. A post 8 feet high casts a shadow 3 ft. long. A church spire at the same time casts a shadow 35 ft. long. How high is the church spire?

3. How long will the shadow of a tree 40 ft. high be when the shadow of a tree 18 ft. high is 12 ft. 5 in. long?

4. How high is a house which casts a shadow 18 ft. long when the shadow of a post $4\frac{1}{2}$ ft. high is 2 ft. 9 in. long?



5. To find the distance from A to the inaccessible point X: Draw AB perpendicular to AX. Join BX. Draw CD parallel to AX. What kind of angles at A and C? What two similar triangles? What sides are measured? What is the proportion? Show how you can find the distance from B to X.

6. Given the lines: $AB = 52$ ft., $BC = 12$ ft., $CD = 20$ ft., to find AX.

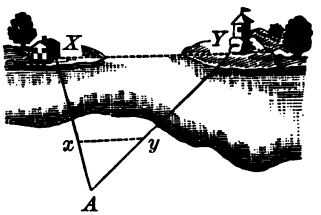
7. Given the lines: $AB = 26$ ft., $BC = 6$ ft., $BD = 10$ ft., to find BX.

8. By drawing other figures from the points A and B, can you find the length of AX?

9. If the point from which distance to X is desired is on the bank of the river, what lines should be drawn?

10. From any point in a lot on one side of the road, find the distance to a point in a lot on the other side of the road without *crossing the road.* Draw diagram illustrating all lines needed for *measurement.*

1. From any point A, to find the distance between two inaccessible points, X, Y: How can the distances AX and AY be found? How can you place the points x and y so that $Ax : AX = Ay : AY$? Why is xy parallel to XY? What two similar triangles? How can XY be found?

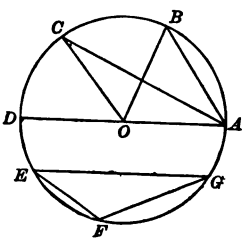


2. If AX is found to be 120 ft. and AY 170 ft., and if Ax is $\frac{1}{10}$ of AX and Ay is $\frac{1}{10}$ of AY, what is the distance xy? What is the distance XY?

3. Make similar problems from actual measurements in a field.

4. Draw a circle. Point out the centre. Observe the distance between the centre and all points of the boundary. Define *circle.*

5. Point out and define: *circumference; diameter; radius; arc; chord; segment; sector; semi-circle.*



6. Draw a circle divided into 4 parts. Each part is a quadrant. Define *quadrant.*

7. Draw a circle divided into 6 parts. Each part is a sextant. Define *sextant.*

8. Draw a circle with a given radius; with a given diameter. Draw a sector with an angle at the centre of 45°. What is the corresponding arc? Draw a sector with an angle at the centre of 80°; of 100°; of 180°. What are the corresponding arcs and segments?

9. Draw a circle. Draw a chord subtending an angle of 60° at the centre. From the centre draw a line perpendicular to this chord. Can you show that the perpendicular line bisects the chord and the arc subtended by it? Can you show that every diameter perpendicular to a chord bisects the chord and also the arcs subtended by it?

10. Can you show that a perpendicular erected at the middle of a chord passes through the centre of the circle?

11. Can you find the centre of a given circle?

12. *Can you find the* centre of a given arc?

1. Can you draw a circle the circumference of which shall pass through any three given points not in a straight line?

2. Draw a chord of 90°. It subtends what arc? The arc is what part of the circumference? Can you inscribe a square in the circle?

3. Draw a chord of 60°. Compare its length with that of the radius. Can you inscribe a regular hexagon in the circle? Can you inscribe an equilateral triangle in the circle?

4. Draw a circle. Draw an angle whose sides are chords and whose vertex is in the circumference. This is an inscribed angle. Can you show that an inscribed angle is equal to one-half of the angle at the centre which has the same arc? Inscribe an angle in a semi-circle. What is the angle?

5. Draw a circle. Draw a line cutting the circumference in two points. This is a secant. Define *secant*. Draw a line touching the circumference in one point without cutting it. This is a tangent. Define *tangent*.

6. Can you show that the tangent is perpendicular to a radius at the point of contact?

7. Can you draw a tangent through a given point in the circumference?

8. Can you draw a tangent through a given point outside of the circumference? How many tangents can be drawn from the same point?

9. Can you inscribe a circle in a given triangle?

10. Can you circumscribe a circle about a given triangle?

11. Can you inscribe a circle in a regular polygon?

12. Can you circumscribe a circle about a regular polygon?

13. Can you with the aid of a protractor and circle draw a regular polygon?

14. Can you without the aid of a protractor draw a regular octagon? a regular hexagon? a regular dodecagon?

15. In what ways can you show the ratio of the circumference of a circle to its diameter? What do you find by measurement? What is the more exact ratio? The number which expresses the *ratio is denoted* by the Greek letter π.

1. Letting $S =$ area or surface, $R =$ radius, and $C =$ circumference, can you show that $S = \frac{1}{2} R \times C$?

2. Can you show that $S = \pi R^2$? **3.** That $S = D^2 \times \frac{\pi}{4}$?

4. Can you show that $C = 2\pi R$?

5. Can you show that $R = \sqrt{\frac{S}{\pi}}$?

6. Can you show that the area of a sector equals one-half the product of its radius by its arc?

7. Can you show how to obtain the area of a segment of a circle ?

8. Can you show how to obtain the area of a circular zone ?

9. Can you show how to obtain the area of a circular ring ?

10. Can you show that the areas of circles are to each other (*a*) as the squares of their diameters ? (*b*) as the squares of their radii ? (*c*) as the squares of their circumferences ?

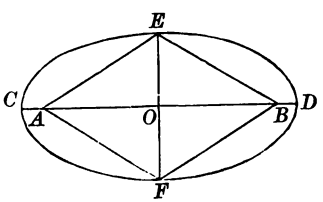
11. The areas of all similar surfaces are to each other as the squares of their homologous sides. Show that this is true in as many cases as you can.

12. Can you show that circumferences of circles are to each other as their radii ?

13. Can you show that areas of two circles are to each other as the squares of their radii ?

14. Can you draw an ellipse ? Can you show that the sum of the distances of every point in the boundary line from two fixed points is the same ? Define ellipse.

15. In the ellipse *CDEF*, point out the major axis; minor axis; foci; eccentricity.



16. Can you show that the sum of the distances of any point of the ellipse from the foci is equal to the major axis ?

17. Can you draw an ellipse when the axes are given? when one axis and the eccentricity are given ?

18. The area of an ellipse is the product of the semi-axes multiplied by π. Can you show this ?

1. Define : *dihedral angle; trihedral angle.*

2. Define : *tetrahedron; pentahedron; hexahedron; octahedron; polyhedron; regular polyhedron.*

3. Define : *prism; right prism; oblique prism; triangular prism; quadrangular prism; parallelopiped; rectangular parallelopiped; cube.*

4. What is the shape of the bases of prisms ? of the sides ?

5. Show how to find the total surface of prisms.

6. Show how to find the volume of a prism.

7. Show how to find the surface of a cube, one edge being given ($S = 6e^2$).

8. Show how to find the volume of a cube, one edge being given ($V = e^3$); the edge of a cube ($e = \sqrt[3]{V}$).

9. Find the total surface and volume of a regular triangular prism, having given : height = 8 in., length of one side = 6 in.; height = 16 ft., length of one side = $12\frac{1}{2}$ ft.

10. Find the total surface and volume of a cube whose edge is 10 in.; whose edge is $6\frac{1}{2}$ ft.; whose edge is 3 ft. 8 in.

11. How high must a prism be whose volume is 1000 cu. in. and the area of whose base is $1\frac{1}{2}$ sq. ft. ?

12. What is the edge of a cube containing 90 cu. ft. ?

13. Define *pyramid; right pyramid; regular pyramid; triangular pyramid; quadrangular pyramid; frustum of a pyramid.*

14. Show how to find the lateral surface of a pyramid.

15. Show how to find the volume of a pyramid.

16. Show how to find the lateral surface of the frustum of a pyramid.

17. Show how to find the volume of the frustum of a pyramid.

18. Find the total surface of a regular octagonal pyramid whose slant height is 16 in. and whose side of base is 6 in.

19. Find the volume of a regular triangular pyramid whose height is 8 ft. and whose side of base is $4\frac{1}{2}$ ft.

20. Find the volume of the frustum of a regular quadrangular pyramid whose height is 10 in. and whose sides of bases are 8 in. and 6 in.

1. Define : *cylinder ; right cylinder.*

2. Show how to find the lateral surface of a cylinder.

3. Show how to find the volume of a cylinder.

4. Define : *cone ; right cone ; frustum of cone.*

5. Show how to find the convex surface of a cone.

6. Show how to find the volume of a cone.

7. Show how to find the surface and the volume of the frustum of a cone.

8. Define : *sphere ; diameter of sphere ; radius of sphere ; great circle ; small circle ; spherical zone ; spherical segment ; spherical sector.*

9. Show how the surface of a sphere is found.

10. Show how the volume of a sphere is found.

11. Find the total surface and volume of a cylinder whose height = 8 ft. and diameter of base = 6 ft. ; whose height = 20 in. and radius of base = 8 in.

12. Find the total surface of a cone whose slant height = 28 in. and diameter of base = 2 ft. ; whose slant height = 6 ft. 3 in. and radius of base = 1 ft. 4 in.

13. Find the volume of a cone whose height = 18 in. and diameter of base = 3 ft. ; whose height = $6\frac{1}{3}$ ft. and radius of base = $2\frac{1}{2}$ ft.

14. Find the volume of the frustum of a cone if the height = 16 in. and radii of bases = $8\frac{1}{3}$ in. and 6 in.

15. Find the surface and volume of a sphere whose radius = 8 in. ; whose diameter = $8\frac{1}{2}$ ft. ; whose circumference = 90 ft.

16. The volume of a cylinder is 80 cu. in. If the diameter is 2 in., what is the height ?

17. The volume of a cone is 6 cu. ft. If the height is 3 ft., what is the area of base ? the diameter of base? the circumference of base ?

18. The volume of a sphere is 600 cu. in. What is the diameter ?

19. The volume of the frustum of a cone is 40 cu. in. ; the radii of bases are 3 in. and 2 in. What is the height ?

20. A cylindrical cistern 6 ft. in diameter is filled with water to the height of $8\frac{1}{2}$ ft. How many gallons ?

1. Compare the size of two cubes, one whose edge is 1 in. and the other whose edge is 2 in.

2. Compare the volume of two cylinders, one whose height and diameter of base are one-half the height and diameter of base of the other.

3. Compare the size of two spheres whose diameters are 2 in. and 4 in. respectively.

4. Point out two solids having exactly the same shape. In these solids, compare the number, shape, and position of faces, and size of polyhedral angles. These solids are similar. Define similar solids. Show in any way you can that similar solids are to each other as the cubes of their corresponding sides.

5. If a cubical vessel whose edge is 1 foot will contain 62.4 lb. of water, what is the weight of the water which fills a cubical vessel whose edge is 2 ft.? whose edge is 3 ft.? What must be the edge of a cubical vessel that will contain 100 lb.? 1000 lb.?

6. A pyramid is cut by a plane parallel to the base and midway between the vertex and the base. Compare the volumes of the entire pyramid and the pyramid cut off.

7. What is the depth of a cubical box which will hold 8 times as much as a similar box whose depth is $2\frac{1}{2}$ ft.?

8. Required the diameter of a sphere which is 64 times as large as a sphere whose diameter is 6 in.

9. How many cubes 3 in. long can be made from a cube 9 in. long?

10. If a conical stack of hay 8 ft. high contains 2 tons, what is the height of a similar stack which contains 8 tons?

11. If a conical bin 4 ft. long contains $12\frac{4}{5}$ bu. of wheat, how much will a bin of the same shape hold that is 9 ft. long? How long will a cubical bin have to be to contain 200 bushels?

12. The volumes of two similar cones are 216 cu. ft. and 1000 cu. ft. If the height of the smaller cone is 8 ft., what is the height of the larger?

13. I have a cistern in the form of a cylinder whose diameter is 3 ft. and whose height is $4\frac{1}{2}$ ft. What must be the dimensions of *a cistern of the* same shape which will hold 3 times as much?

1. A stack of hay in the form of a cone is 12 ft. high and 32 ft. in circumference at the base. What is the diameter? What is the slant height? How far from the top (slant height) must hay be taken to leave half of the stack? to leave $\frac{2}{3}$ of the stack?

2. A large pine tree casts a shadow 48 ft. long. A post 10 ft. high at the same time casts a shadow 6 ft. in length. (*a*) How high is the tree? (*b*) What are the cubic contents of the trunk of the tree if its average circumference is 15 ft.? (*c*) How many cords in the trunk if 25% is allowed for waste?

3. Reckoning the diameter of the earth as 8000 miles, what is the area of its surface? What is the surface of the sun if its diameter is 113 times as great as that of the earth?

4. The diameter of one globe is 8 in.; that of another is 2 ft. The surface of the second globe is how many times as great as that of the first? The volume is how many times as great?

5. The diameter of the moon is .274 as great as that of the earth, and the diameter of the sun is 113 times as great. Compare the size of these three bodies.

6. Find the edge of a cube $\frac{1}{4}$ as large as a cube whose edge is 15 in.

7. A diagonal on one face of a cube is 12 in. Find the surface of the cube. Find the volume.

8. A cone of sugar 15 in. high and 6 in. in diameter is to be divided into two equal parts. How far from the base must it be cut? If it were divided into three equal parts, how would it be divided?

9. The Winchester bushel is $18\frac{1}{2}$ in. in diameter and 8 in. deep. What must be the dimensions of a peck measure of the same shape?

10. How many lead balls .015^{m} in diameter can be made from 50^{Kg} of lead, specific gravity of lead being 11.4?

11. A cube and a sphere have the same surface; viz., 2400 sq. in. What is the difference in cubic contents?

12. An iron ball .072^{m} in diameter weighs 1.8^{Kg}. How heavy is a ball of the same material 1.08^{m} in diameter?

13. What is the volume of a cylinder whose surface is 84^{qm} and whose height and diameter are as 1 : 2?

Practical Measurements.

(For suggestions as to common practice in measurements, see Appendix.)

1. How many feet of lumber are there in a board (*a*) 18 ft. long, 8 in. wide, and 1 in. thick? (*b*) 16 ft. long, 9 in. wide, and $\frac{3}{4}$ in. thick? (*c*) 12 ft. 8 in. long, $8\frac{1}{4}$ in. wide, and 1 in. thick? (*d*) 15 ft. 3 in. long, 10 in. wide at one end, 7 in. wide at the other, and $\frac{7}{8}$ in. thick?

2. How many feet of lumber or board feet are there in a plank (*a*) 10 feet long, 8 in. wide, and 2 in. thick? (*b*) 14 ft. 2 in. long, 7 in. wide, and $1\frac{1}{2}$ in. thick? (*c*) 15 ft. 9 in. long, $9\frac{1}{4}$ in. wide, and $2\frac{1}{2}$ in. thick?

3. Find the cost of 1860 ft. of lumber at \$22.50 per M.

4. How many feet of lumber will it take to make a sidewalk 126 yd. long, 8 ft. 6 in. wide, if the planks used are $1\frac{1}{2}$ in. thick? (No allowance for waste.)

5. At \$18 per thousand, find the total cost of 30 three-by-six joists 18 ft. long; 68 6-inch boards 10 ft. long, $\frac{7}{8}$ in. thick; 24 three-by-four scantlings 15 ft. long.

6. A room $18' \times 10'\ 6'' \times 9'$ has three windows, each $6\frac{1}{2}' \times 3\frac{1}{2}'$, and two doors, each $7\frac{1}{2}' \times 4'$. How many bundles of laths will it take for the walls and ceiling? Cost of lathing the room at 4¢ a square yard?

7. How many bundles of shingles will it take to shingle a double roof 32 ft. 8 in. long, rafters 22 ft. long, shingles to be laid 4 in. to the weather?

8. A house is 48 ft. long, 36 ft. 8 in. wide, and 28 ft. high to the eaves, and the gables are 9 ft. 2 in. high. There are on the gable ends of the house 5 windows, each $5\frac{1}{2}' \times 4'$, and 2 doors, each $8'\ 6'' \times 4'$. On the sides there are 9 windows, each $5\frac{1}{2}' \times 4'$, and 1 door $8'\ 6'' \times 4'\ 6''$. (*a*) Find the number of feet of inch boards for the sides. (*b*) Find the number of feet of inch boards for the gable ends. (*c*) Find the number of feet of $\frac{3}{4}$ in. boards for the double roof. (*d*) Find the cost of shingles for double roof at \$3.80 per thousand. (*e*) Find the cost of shingles for sides and ends at *\$4.20 per thousand.*

1. A barn is 60 ft. by 30 ft., and has a half-pitch roof. How many feet of inch boards in the gable ends? How many feet of lumber in the floor covered with planks $1\frac{1}{2}$ in. thick?

2. How many cords of stone in a pile 12 ft. 8 in. long, 6 ft. 4 in. wide, and 4 ft. 3 in. high?

3. On the supposition that a cord of stone will make 100 cu. ft. of wall, how many cords will it take for a wall 22 ft. 6 in. long, 4 ft. 3 in. high, and 28 in. thick? How many cords are required for the walls of a cellar whose inside measure is 36 ft. 4 in. long, 21 ft. 6 in. wide, and 7 ft. high, the walls being 20 in. thick?

4. How many perches of stonework are there in the walls of a cellar under a building 40 ft. by 20 ft., the walls being $6\frac{1}{2}$ ft. high and 2 ft. thick? Find the answer by exact measurement of corners, and also by taking the outside measure of walls as the length of walls.

5. At \$1.20 a perch, what will it cost to make a wall 6 rd. long, $3\frac{1}{2}$ ft. high, and 30 in. thick?

6. How many cords of stone will it take to build the gable ends and sides of a church 80 ft. long and 45 ft. wide, eaves 20 ft. from the ground, gables 18 ft. high, and walls 24 in. thick? Reckon a cord to make 100 cu. ft., and make no allowance for openings. How much will the walls of the cellar cost at 90¢ a perch, the walls to be 24 in. thick?

7. How many bricks $8'' \times 4'' \times 2''$ in a pile 12 ft. 4 in. long, 5 ft. 4 in. wide, and 4 ft. high? What will they cost at \$14 per M.?

8. Estimate the approximate number of bricks required to build the walls of a house 48 ft. long, 32 ft. wide, 24 ft. high, and 18 in. thick, reckoning 24 bricks to the cubic foot. There are to be 15 windows, each $5\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft., and 3 doors, each $7\frac{1}{2}$ ft. by $5\frac{1}{2}$ ft. In this estimate no allowance is to be made for openings, and the corners are to be counted twice. Estimate the number of bricks that will be required to build the walls, making allowance for openings, and making exact measurements of corners.

9. A cask 2 ft. 9 in. long, having a mean diameter of 18 in., will hold how many gallons of vinegar?

1. A house with gable ends is 60 ft. long, 40 ft. wide, and 25 ft. high to the eaves, the gable being 18 ft. high. There are 18 windows, each 6 ft. by $4\frac{1}{3}$ ft., and 4 doors, each $7\frac{1}{2}$ ft. by $5\frac{1}{2}$ ft. The walls are 12 in. thick. (*a*) By exact measurement, estimate the number of bricks which were required to build the walls, allowing 22 bricks to the cubic foot. (*b*) Find the cost of the bricks at \$9.75 per M. (*c*) How many bundles of shingles did it take to cover the roof, the shingles being laid $4\frac{1}{2}$ in. to the weather? (*d*) How many square feet of boards did it take to lay the double floors, not allowing for partitions?

2. A room is $18\frac{1}{2}$ ft. long, $14\frac{1}{4}$ ft. wide, 9 ft. high. There are 3 windows, each 6 ft. by $4\frac{1}{3}$ ft., and 2 doors, each $7\frac{1}{4}$ ft. by $5\frac{3}{4}$ ft. (*a*) Find the cost of the boards, $\frac{3}{4}$ in. thick, for a double floor at \$15.50 per M. (*b*) Making allowance for $\frac{1}{2}$ of openings, find the cost of lathing the walls and ceiling of the room, reckoning 14 laths to the square yard, the cost of laths 50¢ a bundle, and the cost of laying 40¢ a thousand. (*c*) Cost of plastering at 18¢ a square yard, allowing one-half the area of the openings. (*d*) Estimate the approximate number of rolls of paper that will be required to cover the walls, allowing 3 rolls for waste in laying. (*e*) How many yards of carpeting 27 in. wide will be required for the floor?

3. What is the cost of a rug $3\frac{1}{2}$ yd. by 3 yd. at 75¢ a square yard, together with a border 18 in. wide at $62\frac{1}{2}$¢ a yard, the corners for the border to be counted twice, on account of waste in making?

4. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for the floor of a room 15 ft. 3 in. long, 12 ft. 6 in. wide, if the strips are laid lengthwise, and there is a waste of 4 in. in each strip in matching the pattern? How many yards are required if the strips are laid across the room?

5. Supposing that the above room is 9 ft. 6 in. high, how many rolls of paper will be required for the walls, making no deductions for openings? At 30¢ per square yard, how much will it cost to plaster the walls and ceiling, no allowance being made for openings? *At 6¢ per square* yard, what will it cost to kalsomine the ceiling?

1. About how many tons of hay in a mow 42 ft. long and 18 ft. wide, the hay having an average depth of 10 ft.?

2. Find the approximate amount of hay in a conical stack whose height is 18 ft. and diameter of base is 15 ft.

3. Find the approximate amount of coal that can be put into a bin 8 ft. long, 6 ft. wide, and 4 ft. deep.

4. How deep will coal have to be in a bin 8 ft. square to contain 20 tons?

5. How high must wood be piled in a car 36 ft. long, $7\frac{1}{2}$ ft. wide, to contain 20 cords?

6. Find approximately the number of cords of stone it will take to build a wall 1 mile long, 4 ft. high, and 2 ft. thick. About how much lime and sand will be required to build the wall?

7. About how many gallons of water can be put into a cubical tank whose edge is 10 ft.? About how much into a circular tank 10 ft. high and 10 ft. in diameter?

8. About how many barrels of water will a cistern hold which is in the form of a frustum of a cone whose height is 8 ft. and the diameter of bases 6 ft. and 10 ft.?

9. How many pounds of fourpenny nails were required to lay the shingles for the house described on page 62, allowing 6 lb. for laying 1000 shingles?

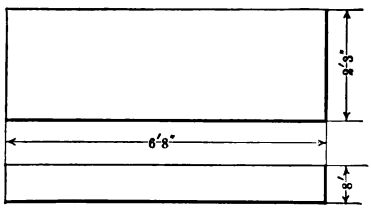
10. In a wood-lot there were counted 384 trees whose average height was estimated to be 40 ft. and average diameter 8 in. About how many cords of wood and logs in these trees? Other wood on the lot was estimated to amount to 30 cords. If wood and logs are worth \$3.50 a cord standing, about how much may be paid for the lot, considering the land to be worth \$120, and allowing a margin of profit of 25%?

11. About how many tons of ice can be put into an ice-house 40 ft. long, 30 ft. wide, and 20 ft. high, specific gravity of ice being .93? How many square yards of surface will have to be cut to get this amount, at an average thickness of 9 in.?

12. About how much will it cost to lay the shingles referred to in *Ex. 9, p. 60, a day's* work being 1800 for a man, at \$2.50 a day?

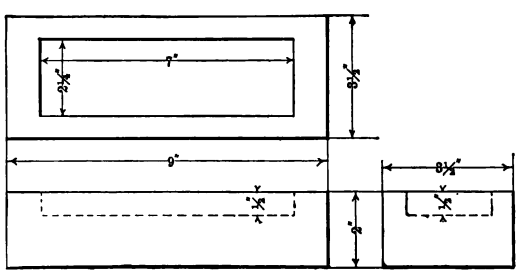
Working Drawings.

1. I desire to have a stone door-step cut 6 ft. 8 in. long, 2 ft. 3 in. wide, and 8 in. thick. This is a working drawing such as would be given to the stone-cutter. Estimate the cost of it at 60¢ a cubic foot.



2. A man orders 2 blocks made in the shape of a rectangular solid, each 1 ft. 8 in. long, 1 ft. 2 in. wide, and 10 in. high. Make a working drawing such as he would give the carpenter, and find the number of feet of lumber (board feet) in the blocks.

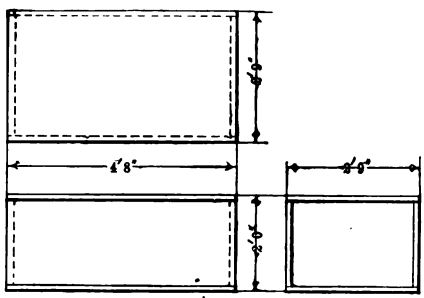
3. A man wishes to have made 100 blocks for oil-stones, each 9 in. long, $3\frac{1}{2}$ in. wide, and 2 in. thick. The mortise for the stone is to be 7 in. long, $2\frac{1}{4}$ in. wide, and $\frac{1}{2}$ in. deep. This cut represents a working drawing such as is given to the carpenter. Find the number of square feet of lumber required to make the blocks, allowing as much material for waste as is used in the blocks.



4. I wish to have made a drinking-trough hewed from a solid block. The outside measure of the trough is to be 8 ft. 6 in. long, 2 ft. 9 in. wide, and 20 in. thick, and the place for holding the water is to be 7 ft. 6 in. long, 2 ft. 1 in. wide, and 1 ft. deep. Make a working drawing, and find the cost of lumber required at $3\frac{1}{2}$¢ a foot, allowing as many feet of waste as is used in the trough.

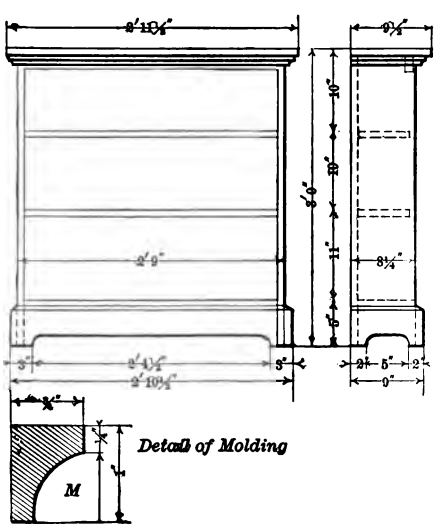
5. A trough 6′ 3″ long, 1′ 8″ wide, 15″ deep is ordered. Planks *2″ thick.* *Make* drawing, and find feet of lumber.

1. I order of a carpenter a packing-case 4 ft. 8 in. long, 2 ft. 9 in. wide, and 2 ft. high, the sides and cover to be fitted as in the accompanying drawing. The lumber in the ends is to be $1\frac{1}{2}$ in. thick, and in all other parts 1 in. thick. Find the number of square feet of lumber required to make the box. Find the cost at $1\frac{3}{4}¢$ a square foot and \$3.50 for making. How many square feet of paper will it take to line the inside of the box? How many packages of tacks can be put into the box, each package being $3\frac{5}{16}$ in. long, $2\frac{7}{32}$ in. wide, and $1\frac{7}{16}$ in. thick?



2. A man having 20 M. of box-boards desires to have them made into boxes 34 in. long, 22 in. wide, and 18 in. high. Make a working drawing of the box desired, and find how many boxes can be made, allowing 5% waste of boards in making.

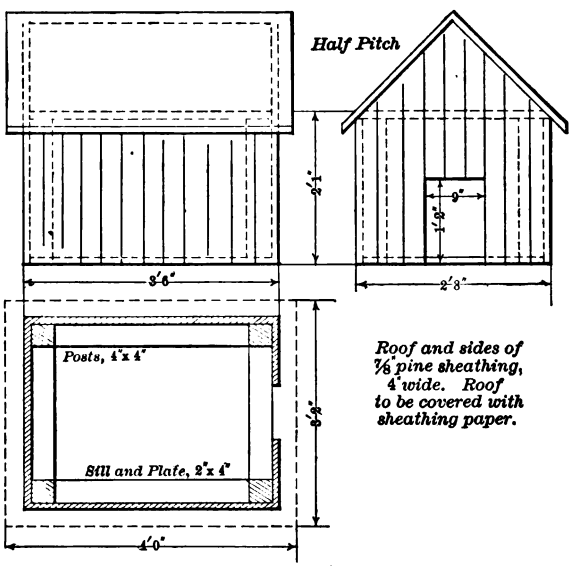
3. I wish to order a book-case such as is indicated in the accompanying drawing. Estimate approximately the number of feet of lumber required, the boards to be $\frac{3}{4}$ in. thick. Allow $\frac{1}{4}$ of lumber for waste. Find cost of book-case, lumber being $2\frac{3}{4}¢$ a foot, and \$10 for making.



4. How many ft. of lumber in your teacher's desk? Make drawing.

1. Make a working drawing such as you would make in ordering a plain square table with given dimensions, and find the number of square feet of lumber that will be required.

2. How much lumber, board measure, will be required to make a poultry house of the shape and dimensions given in the following working drawing, allowing one-fourth for waste? How many



sheathing-boards on each side? How many on each end? How many square feet of sheathing-boards, allowing 20% waste? What will the sheathing-paper for the roof cost at 4¢ a square yard?

3. Make as nearly as you can to scale a plan of the ground floor and each elevation of your school-house, and, if it is made of wood, estimate the amount of lumber required to build it. If made of brick, estimate the number of bricks and amount of lumber required to build it.

4. Make a working drawing such as you would make in ordering *a clothes-press,* and compute the cost of it.

Miscellaneous.

1. How many square feet are there in a circle whose diameter is 4 ft. 3 in.? in a circle whose diameter is 8 ft. 9 in.?

2. Draw a circle whose radius is 4 in. How many square inches in the circle? If this circle represents a lot of land drawn on a scale of $\frac{1}{2}$ in. to a rod, how many square rods in the lot?

3. What is the diameter of a circle whose area is 1 A.? What is the circumference of a circle whose area is 3 A. 40 sq. rd.?

4. What is the area of a sector whose arc is 90° in a circle having a diameter of 10 ft.? What is the area of a sector in the same circle whose arc is 50°? whose arc is 20°? whose arc is 80°?

5. A square field is 40 rods long. What is the length of a square field 4 times as large?

6. A rectangular cistern is 9 ft. 9 in. long, 6 ft. 6 in. wide, and $4\frac{1}{2}$ ft. deep. Find the expense of lining the sides and bottom with sheet lead weighing 8 lb. per square foot, at 6¢ per pound.

7. How many feet of boards $\frac{7}{8}$ in. thick will be required to make a cubical box whose edges measure 1 ft. outside?

8. The dimensions of a rectangle are 48 yd. and 36 yd. Find its diagonal. Find the diagonal of a square of the same area.

9. The hypothenuse of a right triangle is 50 ft., and one angle is 45°. Find the other sides.

10. The base and perpendicular of a right triangle are 120 ft. and 90 ft. respectively. If a perpendicular be dropped from the vertex of the right angle to the hypothenuse, find the length of the parts into which the hypothenuse is divided and the area of each part of the triangle.

11. The circumference of a dome in shape of a hemisphere is 94.248 ft. How many square feet of tin roofing will cover it?

12. If the ball on the top of St. Paul's Cathedral is 6 ft. in diameter, what will it cost to gild it at 7¢ per square inch?

13. If a cubic inch of iron weighs $4\frac{1}{2}$ oz., what will be the weight of a 6 in. cannon ball?

14. The base of a right triangle is 30 ft. and the opposite angle is 30°. What is the perpendicular?

1. A tree is broken by the wind 20 ft. from the ground, and the tip touches it so as to make an angle of 30°. What is the entire height of the tree?

2. Find the area of a rhombus whose diagonals are 80 yd. and 60 yd. respectively.

3. Find the area of a rhomboid whose sides are 60 ft. and 40 ft., and one of whose angles is 45°.

4. The diagonals of a rhombus are 80 yd. and 60 yd. Find each side and altitude.

5. The diameters of two spheres are 6 in. and 12 in. respectively. What is the ratio of their surfaces? of their volumes?

6. A 9 in. ball was turned down so that it makes a 6 in. ball. What part was cut away?

7. The diameter of a ball is 8 in. What is the diameter of a similar one weighing $\frac{1}{64}$ as much?

8. The three dimensions of a box are each doubled. To what extent is its capacity affected?

9. The inside diameter of a hemispherical dome is 10 ft., and the uniform thickness is 2 ft. If it costs \$3 to paint the inside, what ought it to cost to paint the outside?

10. The hour-hand of a clock is 4 in. long. How far does it move in 1 hour?

11. What is the radius of a circle whose area is $\frac{1}{4}$ A.?

12. What is the area of a semicircle whose radius is 10 ft.?

13. The diameter of a circle is 20 ft. What must be the side of a square of equal area?

14. The diameter of a circle is 30 ft. What is the diameter of a circle twice as large?

15. A circle, a square, and an equilateral triangle have each a perimeter of 144 ft. Compare their areas.

16. Find the cost of concreting a circular plot 42 ft. in diameter at 60¢ per square yard, leaving in the centre a hexagonal space, each of whose sides measures 3 ft.

17. The height of the frustum of a cone is $\frac{1}{4}$ the height of the *entire cone.* Compare the volumes of the entire cone and frustum.

1. Find the surface of a sphere whose diameter is 40 ft.; 60 ft. Find the surface of a sphere whose volume is 1000 cu. in.; 50 cu. ft.

2. What is the diameter of a circle having the same area as a triangle whose base is 8 ft. and altitude 10 ft.?

3. How many square feet in the surface of a spire which is in the form of a hexagonal pyramid 80 ft. high, each side of base 12 ft.?

4. A cylindrical reservoir 18 ft. 6 in. in diameter has a capacity of 3,800,000 gallons. What is the height?

5. If a stack of hay 12 ft. in height weighs 16.7 tons, what is the weight of a similar stack that is 8 ft. high?

6. If a stone $6\frac{1}{2}$ ft. long, 20 in. wide, and 12 in. thick weighs 1711 lb., what will be the dimensions of a similar stone that weighs 6 T.?

7. The Great Pyramid of Egypt was 480 ft. 9 in. high, and its base was 764 ft. square. What was the area of its surface? What was its volume in tons?

8. Find the number of shingles 4 in. wide to cover 1 sq. ft. if laid 3 in. to the weather; if laid $4\frac{1}{2}$ in. to the weather. Allowing nothing for waste, how many bundles of shingles laid $4\frac{1}{2}$ in. to the weather will be required to cover the pitched roof of a building 38 ft. long, the width of each side of the roof being 26 ft. 6 in.?

9. I have two square lots of land, the ratio of whose areas is as 3 to 5. If the larger lot contains $\frac{3}{4}$ A., what is the length of a side of the smaller lot?

10. Find the side of a square field which is the same size as a field 2.18 Km long and 480 m wide.

11. The longest side of a piece of land in the shape of a right-angled triangle cannot be measured on account of a swamp. What is its length, if the lengths of the other two sides are 48 rd. 5 ft. and 42 rd. 10 ft.?

12. Two persons start from the same place, one walking due north at the rate of 3 miles an hour, and the other due east at the rate of 4 miles an hour. How far apart will they be at the end of 2 hours? If they both walked at the rate of 4 miles an hour, in how *long a time* would they be 20 miles apart?

1. How many cubic inches of lead would be required to make 1000 half-inch bullets ?

2. If a round pillar 8 in. in diameter has 6 cu. ft. of stone in it, what is the diameter of a pillar of equal height that contains 40 cu. ft. ?

3. How much surface of gold foil beaten into sheets of .06 in. thick will 1 ounce cover ?

4. I have a rectangular lot of land 350 ft. long, 180 ft. wide. How far is it from the centre of this lot to each of the four corners ?

5. What is the difference in length between a fence around the above lot and one around a circular lot of the same size ?

6. A church steeple is in the form of a pyramid. Its base is a square 10 ft. on each side, and its slant height is 80 ft. What is the cost of painting it at 25¢ per square yard ?

7. How long must wire .03 in. in diameter be to weigh a pound, specific gravity being 7.6 ? How many feet of the same wire will a cubic foot make, allowing no loss for waste ?

8. How high on the stakes must wood be piled in a cart 10 ft. long, 3 ft. 6 in. wide, to contain a cord, the wood being of usual length, piled lengthwise in two tiers with a tier piled crosswise between them ?

9. What is the length of a chord which cuts one-third of a circle whose diameter is 18 in. ?

10. A man wishes to make a circular race-course 80 ft. wide, exactly one-half of a mile long, measured in the centre of the track. Required the radius of the circular area inclosed, and also the area of the race-course.

11. Around the outside of an elliptical shaped garden whose greater diameter is 60 ft. and whose lesser diameter is 48 ft., there is a walk 6 feet wide. What is the area of the garden ? What is the area of the walk ?

12. Estimate the amount of lumber required for all the frame-work of the house in which you live; for all the flooring; for the *boards* which cover the roof, ends, and sides; bundles of shingles *required; bundles* of clapboards.

SECTION V.

BUSINESS EXERCISES AND BOOK-KEEPING.

1. Write a receipt for money received; for other things received. State the necessary parts to a receipt. Define receipt.

2. Write a due-bill for money; for goods.

3. Define debtor; creditor. Write a bill of sale receipted by a clerk. Write a bill for services. Write a statement. Write a combined statement and bill. What are the necessary parts of a bill? Define bill.

4. Write an order for money; an order for goods. Explain the use of the terms "To order," "To bearer." How does a bank check differ from an ordinary order? What are the necessary parts of an order or check? Explain two ways of indorsing an order or check.

5. Define the following terms: draft; bills receivable; bills payable; list price; net price; discount. Write and explain a bill containing two trade discounts.

6. Give examples of marking goods so as to denote the cost and selling price. Compute the per cent of gain or loss.

7. Define interest; compound interest; accurate interest.

8. What is a promissory note? What are the necessary parts of a note? Who is the maker? Who is the payee? What different kinds of notes? Explain in full the difference between a negotiable note and a non-negotiable note. What is an indorsement? Describe the different kinds of indorsements. Explain "Without recourse." What is the day of maturity? What are days of grace? What is a joint note? What is a joint and several note? What is a collateral note? What is an accommodation note? Give examples of all the above points.

1. What parties are liable for the payment of a note, and under what circumstances? What kind of notes allow no days of grace for payment? What is done if a note matures on Sunday or on a legal holiday? If no time is given in the note, when is it payable? What notes do not bear interest? A note not specifying that interest will be paid, and due at a definite time, bears interest when? When is a note void?

2. Give example of a note with partial payments.

3. Explain the purpose and working of banks for circulation and deposit; of savings banks; of coöperative banks. How are national banks started? How is money deposited and withdrawn? Describe national bank notes of various kinds. Describe the method of discounting notes at the bank. What is done by the bank if the money is not paid when due?

4. Tell all you can about mortgages and mortgage notes.

5. Describe all ways of sending money from one place to another in this country. Define how drafts are made and sent to a foreign country. Define drawer; drawee; payee. Explain the difference between a sight draft and a time draft. How is a draft accepted? Give an example of sending a bill of exchange to England; to France; to Germany.

6. Describe in full the buying and using of a letter of credit. What advantages has it over a draft? What is a circular note?

7. Give the purposes of taxation, and explain all the different methods of taxation employed.

8. Show by example how the rate of taxation in a city or town is determined. Make out a table for the convenience of assessors in making out the tax bills. From this table compute the taxes of certain property owners. Compute the amount of property taxed from the amount of tax.

9. Find the rate of duty upon certain articles, and make problems such as might occur in importing them.

10. Give the different kinds of insurance. Define policy; premium; underwriter. Give and explain a problem involving *insurance and loss* by fire.

1. Show how a stock company is formed and how stock certificates are made? Who are the shareholders? Who are the board of directors? Explain the following terms, and give examples: dividend; assessment; par value; premium.

2. What are bonds? Tell all you can about various kinds of government bonds. Explain the difference between stocks and bonds. What is a coupon? Explain the quotations of stocks and bonds in a recent newspaper.

3. Give examples of business done with a broker in making investments. Show the loss or gain in selling.

4. You buy of James Scott a piece of land for a certain sum of money, paying part cash and giving a note for the balance, due in 1 year with interest. Before the note matures you make two payments. Write the note with indorsements, and estimate the balance due at maturity of the note.

5. Write an order for books such as might be sent to Ginn & Co. Write a bill such as they would send in return. Write a check for payment. What would Ginn & Co. do with the check? What becomes of the check?

6. William Robinson, grocer, of Yonkers, N. Y., orders from Stacy & Co., of New York, groceries. Stacy & Co. send the goods with bill, with two trade discounts, payment due in 30 days, with discount of $2\frac{1}{2}\%$ for cash. Write the order and the bill. Robinson decides to borrow the money at a bank, and send the money. Write the note that Robinson gives the bank. Write the cashier's check that Robinson sends to Stacy & Co. Suppose Robinson fails in business before the note is due, what is done? On the supposition that no indorser was required, and that Robinson pays 42 cents on the dollar, what sum does the bank collect? How much, if anything, do Stacy & Co. lose by the failure?

7. Give examples of different kinds of partnership, and give a problem involved in the partnership of three persons with different amounts of capital invested.

8. Define: insolvency; assets; liabilities; net capital. Give an example of insolvency and payment to creditors.

The following outline indicates the cash transactions of a country hardware merchant:

Sept. 1

Balance, $200.00

2

Sell, —

8 lb. nails,	.06
7 " "	.04
1 carpet sweeper,	2.38

4

Buy, —

3 lb. cheese,	.18
2 lb. Boston biscuit,	.14
1 gal. syrup,	.85
3 doz. oranges,	.42

5

Sell, —

3 chisels,	.65
4 corn brooms,	.28
3 wooden rakes,	.45
2 iron rakes,	.42

6

Buy, —

2 lb. Java Coffee @	.34
3 lb. Formosa Tea @	.75
2 cans Cal. pears @	.35
1 bbl. St. Louis flour,	5.75

7

Balance and go on.

9

Sales to date, 43.76

10

Pay store rent, 62.50

11

Sell, —

2 clothes driers,	1.25
3 doz. lamp chimneys	.95
6 balls twine,	.38
4 spades,	1.12

12

Pay gas bill, 9.35

13

Sell, —

2 doz. hoes,	3.75
1 " Dover egg-beater,	1.50

14

Buy 1 pair rubber boots, 1.87

16

Sell, —

2 shovels,	1.15
2 doz. quart cans,	1.83
3 large tubs,	.92

17

Buy, —

15 bu. oats,	.55
1 ton Eng. hay,	21.75

18

Sales to date, 85.60

19

Buy, —

3 pair overalls,	.75
1 rubber coat,	5.25

20

Sell 3 coal sifters, .85

On the following page the first part of this account appears as *it is written* by the merchant in his cash book.

Cash.

Sept. 1	To balance,		200	00	Sept. 4	By 3 lb. cheese,	.18		54
" 2	" 8 lb. nails,	.06		48	" "	" 2 " Bost. biscuit,	.14		28
" "	" 7 " "	.04		28	" "	" 1 gal. syrup,			85
" "	" 1 carpet sweeper,		2	38	" "	" 3 doz. oranges,	42	1	26
" 5	" 3 chisels,	.65	1	95	" 6	" 2 lb. Java coffee,	.34		68
" "	" 4 corn brooms,	.28	1	12	" "	" 3 " Formosa tea,	.75	2	25
" "	" 3 wooden rakes,	.45	1	35	" "	" 2 cans Cal. pears,	.35		70
" "	" 2 iron "	.42		84	" "	" 1 bbl. St. L. flour,		5	75
					" 7	" balance,		196	09
			208	40				208	40
Sept. 7	To balance,		196	09					

Copy this account, and explain each item. Continue the account as indicated in the outline, and strike a balance Sept. 21.

The following outline indicates daily entries of a cash account. Begin with the first day of the present month, and make entries as indicated for every day except Sunday. Balance the account after the last entry :

Cash on hand, $192.00

Buy, —
100 bbl. apples, 1.60
200 lb. Vt. butter, .24

Sell, —
3 gal. P. R. molasses, .58
12 lb. Mocha coffee, .32

Pay int. on $300, 5 mo. 15 da.

Sell, —
25 bbl. apples, profit, 40%.
50 lb. Vt. butter, profit, 25%.
3 bottles chow-chow, .85

Pay water bill, 12.00

Sell, —
40 bbl. apples, profit, 25%.
60 lb. Vt. butter, profit, 33⅓%.
5 cans Cal. peaches, .35

Collect int. on $240, 6 mo. 10 da.

Sales to date, 43.76

Buy 2 cd. oak wood, 5.33

Pick over my apples and find 20% of them worthless.
Sell 20 bbl. at a profit of 30% and the rest at cost.

Balance and go on.

I keep 20 lb. butter for home, and sell the balance, profit, 10¢.

Pay gas bill, 7.20

Sell, —
2 bbl. St. Louis flour, 5.25
3 quarts cranberries, .13
6 cans tomatoes, .16
3 lb. currants, .18

Give a poor family, 10.00

Sell, —
2½ lb. sage cheese, .18
8 doz. R. I. eggs, .24

From the following outline write cash account. Write also the accounts of A and B, using any names you please :

Cash on hand,	$230.00
A owes me,	53.70
I owe B,	12.75
A works for me 5 days,	1.75
And I sell him, —	
2 lb., Formosa tea,	.63
3 " Rio coffee,	.32
4 " Malaga raisins,	.18
Sell, —	
3 bbl. Minn. flour,	5.75
25 lb. B. H. sugar,	.05
10 gal. vinegar,	.20
Buy of B, —	
3 pairs brogans,	1.75
2 " ladies' slippers,	1.25
Buy, —	
3 boxes steel pens,	.62
2 reams note paper,	2.50
5 pkg. stamped envelopes,	.75
Sales to date,	22.15
A brings me, —	
3 loads gravel,	.75
And buys, —	
1 bbl. St. Louis flour,	6.13
2 gal. N. O. molasses,	.58
3 bags *Indian meal*,	2.17
Sell B, 2 bu. N. S. potatoes,	.92
And buy of him, —	
3 pairs children's shoes,	2.13
5 " rubbers,	.65
2 " calf boots,	3.25
Pay blacksmith's bill,	23.50
Sell : 15 lb. Rangoon rice,	.09
26 gal. kerosene,	.11
8 lb. Dutch cheese,	.22
23 pkg. flower seeds,	.08
Sell A : 17 lb. buckwheat,	.05
2 doz. eggs,	.24
Buy of B, —	
2 pairs ladies' gaiters,	3.63
1 pair rubber boots,	3.25
Sell : 6 lb. Vt. butter,	.28
7 lb. sage cheese,	.18
3 gal. syrup,	.75
Buy 1 wheelbarrow,	2.50
Sell : 3 lb. crackers,	.14
12 lb. Mocha coffee,	.30
3 boxes sardines,	.63
5 " mustard,	.25
4 doz. lemons,	.28
Sales to date,	[illegible]

1. From the following items of daily transactions write cash account; also the accounts of A and B, using any names you please:

Cash on hand,	$137.50
A owes me,	28.75
I owe B,	13.50
Sell, —	
3 bu. apples,	.80
2 bbl. potatoes,	1.85
50 lb. B. H. sugar,	.04
Buy of A, —	
3 loads loam,	1.25
6 " gravel,	.88
3 cd. oak wood,	5.25
A pays me,	12.75
And works 3 days,	1.75
Sales to date,	62.50
I pay B,	13.50
Buy, —	
28 gal. vinegar,	.12
12 bu. oats,	.55
Sell, —	
8 lb. Formosa tea,	.62
12 lb. Mocha coffee,	.30
3 doz. bananas,	.42
Pay gas bill,	7.38
Buy of A, —	
2 tons Eng. hay,	18.75
3 cd. chestnut wood,	5.25
And	
I lend him,	5.00
Sales to date,	27.90
Pay tax bill,	27.62
Sell, —	
6 gal. N. O. molasses,	.62
22 lb. St. Albans butter,	.30
3 " raisins,	.18
5 " Dutch cheese,	.23
6 doz. Florida oranges,	.50

2. What account books have you used in this section?

3. Show by example the advantage of having a ledger in which accounts may be posted.

4. In making a cash account, what items are placed on the debit side? What items are placed on the credit side?

5. *What items* are placed in the ledger? When are they *written?* When is a trial balance made? What does it show?

Notes of transactions for day book and ledger :

Mdse. on hand, $1243.65
Cash " 50.00
A owes me on $^{a}/_{c}$ 44.06
I owe B " 18.00

I sell C on $^{a}/_{c}$ —
4 bbl. St. Louis flour, 8.50
10 lb. Rio coffee, .35

Sell for cash : 3 doz. eggs, .28
2 lb. rice, .10
5 gal. kerosene, .18
2 lb. raisins, .20

Buy of C : 25 yd. sheeting, .09
12 yd. drilling, .15
And I pay him, 15.00

A pays me, 20.00

Sell C : 2 boxes mustard, .25
2 gal. N. O. molasses, .65
5 lb. best currants, .08
4 cans best Lima beans, .16
And
He works for me 2 days, 1.75
And furnishes, —
200 ft. sodding, .02

Buy for cash, —
1 pair brogans, 2.25
2 cords wood, 5.75

Sell for cash : 2 lb. tapioca, .08
2 boxes gelatine, .16
10 lb. boneless cod, .08

Sell A : 2 lb. Cayenne pepper, .45
2 lb. Eng. breakfast tea, .75
2 " pimento, .25

Sell C : 10 lb. Rangoon rice, .07
20 lb. Scotch oatmeal, .04
And he pays me, 8.00

A pays me, 20.00

Pay tax bill, 30.00

C works for me 3 days, 1.75

Sell C, —
10 pkg. yeast powder, .10
25 lb. salt pork, .12
$\frac{1}{2}$ " nutmeg, 1.10
And he sells me, —
2 cd. chestnut wood, 6.75

Sell for cash : 5 lb. prunes, .10
3 lb. saleratus, .08
3 boxes blacking, .15

A's account is settled with cash.

Mdse. on hand at the end of the month, 1400.00

A form of day book and ledger for the above transactions is given *on the following four* pages. Copy and complete the accounts.

Day Book.

West Newton, January, 1, 1894. 1

		Mdse. on hand,		*1243*	*65*		
C.B.		*Cash* "		*50*	*00*		
	3	*Amos Lawrence owes me on* a/c		*44*	*06*	*1337*	*71*
	4	*I owe Hamilton Brown,* "				*18*	*00*
		2					
	4	*Charles Smith,*	*Dr.*				
		To 4 bbl. St. Louis flour,	*8.50*	*34*	*00*		
		" *10 lb. Rio coffee,*	*.35*	*3*	*50*	*37*	*50*
		5					
	4	*Charles Smith,*	*Cr.*				
		By 25 yd. sheeting,	*.09*	*2*	*25*		
		" *12* " *drilling,*	*.15*	*1*	*80*	*4*	*05*
		Dr.					
C.B.	*4*	*To cash on* a/c				*15*	*00*
		7					
	3	*Amos Lawrence,*	*Cr.*				
C.B.		*By cash on* a/c				*20*	*00*
		8					
	4	*Charles Smith,*	*Dr.*				
		To 2 boxes Keen's mustard,	*.25*		*50*		
		" *2 gal. N. O. molasses,*	*.65*	*1*	*30*		
		" *5 lb. best currants,*	*.08*		*40*		
		" *4 cans best Lima beans,*	*.16*		*64*	*2*	*84*
		Cr.					
	4	*By 2 days' work,*	*1.75*	*3*	*50*		
		" *200 ft. sodding,*	*.02*	*4*	*00*	*7*	*50*

Day Book.

2

West Newton, January 12, 1884.

	3	*Amos Lawrence,*	*Dr.*				
		To 2 lb. Cayenne pepper,	*.45*		*90*		
		" *2* " *English breakfast tea,*	*.75*	*1*	*50*		
		" *2* " *pimento,*	*.25*		*50*	*2*	*90*
		13					
	4	*Charles Smith,*	*Dr.*				
		To 10 lb. Rangoon rice,	*.07*		*70*		
		" *20* " *Scotch oatmeal,*	*.04*		*80*	*1*	*50*
		Cr.					
C.B.	4	*By cash, on* $^{a}/_{c}$				*8*	*00*
		15					
	3	*Amos Lawrence,*	*Cr.*				
C.B.		*By cash, on* $^{a}/_{c}$				*20*	*00*
		17					
	4	*Charles Smith,*	*Cr.*				
		By 3 days' work,	*1.75*			*5*	*25*
		18					
	4	*Charles Smith,*	*Dr.*				
		To 10 pkg. Horsford's Yeast Powder,	*.10*	*1*	*00*		
		" *25 lb. salt pork,*	*.12*	*3*	*00*		
		" $\frac{1}{2}$ " *best nutmegs,*	*1.10*		*55*	*4*	*55*
		Cr.					
	4	*By 2 cd. chestnut wood,*	*6.75*			*13*	*50*
		25					
C.B.	3	*Amos Lawrence,*	*Cr.*				
		By cash in full,				*6*	*96*

Cash Book and Ledger.

3

Dr.		Cash.				Cr.			
Jan. 1	To balance,		50	00	Jan. 1	By C. Smith, on a/c	1	15	00
" 4	" 3 doz. eggs,	.28		84	" 10	" 1 pr. brogans,		2	25
" "	" 2 lb. rice,	.10		20	" "	" 2 cd. wood,	5.75	11	50
" "	" 5 gal. kerosene,	.18		90	" "	" tax bill, 1883,		30	00
" "	" 2 lb. raisins,	.20		40	" 31	" balance,			
" 7	" A. Lawrence, on a/c	1	20	00					
" 11	" 2 lb. tapioca,	.08		16					
" "	" 2 boxes gelatine,	.16		32					
" "	" 10 lb. boneless cod,	.08		80					
" 13	" C. Smith, on a/c	2	8	00					
" 15	" A. Lawrence, on a/c	2							
" 20	" 5 lb. prunes,	.10		50					
" "	" 3 " saleratus,	.08		24					
" "	" 3 boxes blacking,	.15		45					
" 25	" A. Lawrence,	2							
Feb. 1	To balance,		51	02					

Dr.		Amos Lawrence.				Cr.			
Jan. 1	To balance,	1	44	06	Jan. 7	By cash on a/c	1	20	00
" 12	" mdse.,	2			" 15	" " "	2		
					" 25	" " in full,			

4

Dr. HAMILTON BROWN. **Cr.**

						Jan. 1	*By balance,*	*1*	*18*	*00*

Dr. CHARLES SMITH. **Cr.**

Jan.	*2*	*To mdse.,*	*1*	*37*	*50*	*Jan.*	*5*	*By mdse.,*	*1*	*4*	*05*
"	*5*	" *cash on* a/c	*1*	*15*	*00*	"	*8*	" *sundries,*	*1*	*7*	*50*
"	*8*	" *mdse.,*	*1*	*2*	*84*	"	*13*	" *cash on* a/c	*2*	*8*	*00*
"	*13*	" "	*2*			"	*17*	" *labor,*	*2*		
"	*18*	" "	*2*			"	*18*	" *wood,*	*2*		
						"	*31*	" *balance,*			
Feb.	*1*	*To balance,*		*23*	*09*						

Dr. BALANCE SHEET. **Cr.**

Jan.	*31*	*To mdse. on hand,*		*1400*	*00*	*Jan.*	*31*	*By H. Brown,*			
"	"	" *cash* "									
"	"	" *C. Smith,*				"	"	" *N. C.,*			

My N. C. Jan. 31 is $
" " " *1 was* $
My Net Gain is $

From the following notes of transactions, make in proper form day book and ledger accounts:

Mdse. on hand, $500.00
Cash " 42.00

Sell A. A. Evans, —
15 gal. N. O. molasses, .60
4 lb. Formosa tea, .75

Sell for cash: 1 doz. oranges, .25
3 doz. lemons, .20

Sell Hiram Carter, —
2 lb. Vt. butter, .25
4 " Persian dates, .15
And he lends me, 5.00

A. A. Evans pays me, 6.00
And I sell him, —
2 bu. potatoes, .80

Sell Hiram Carter, —
3 lb. Rio coffee, .25
1 box sardines, .50
3 lb. Smyrna figs, .20
10 lb. B. H. sugar, .08
And
He works for me 2 days, 1.87
And brings me, —
2 loads gravel, .63

Buy for cash: 10 yd. calico, .10
15 yd. drilling, .12
20 " sheeting, .08

Sell A. A. Evans, —
2½ lb. Mocha coffee, .32
8 " *Rangoon* rice, .10
4 " *Malaga* raisins, .18

Sell Hiram Carter, —
3 lb. currants, .15
1 bbl. St. Louis flour, 6.75
And
He pays me on a/c 15.00
And brings me, —
1½ cd. chestnut wood, 5.00

A. A. Evans pays me, 20.00

Pay cash for, —
4 pairs brogans, 2.25

Hiram Carter works for me
2 days, 1.38

Sell David Grant, —
2 doz. bananas, .50
3 gal. P. R. molasses, .65
And he brings me, —
25 gal. vinegar, .19

Sell for cash, —
2 lb. cheese, .20
3 " nails, .12
4 gal. oil, .15
3 boxes mustard, .22
2 large tubs, .92

Hiram Carter received from
me on a/c 12.50

Sell for cash: 2 bottles olives, .83
6 lb. lard, .13
3 doz. eggs, .24
2 gal. syrup, .72

Mdse. on hand, 585.00

From the following items make day book and ledger accounts :

Cash on hand, $296.45
Mdse. " 1275.90
I owe A, 75.67
C owes me, 29.48

Sell A, —
50 yd. French print, .22

Buy of C, —
250 ft. pine lumber, .08

Pay A, 10.00

Lend C, 44.52

Sell C,
70 yd. Waltham sheeting, .09

Cash sales to date, 51.17

C works for me 4 days, 2.00

Sell A, —
25 yd. broadcloth, 2.75
100 spools Clark's cotton, .05

Buy for cash, —
50 lb. Y. H. tea, .60
2 doz. bananas, .42
10 boxes sardines, .75
10 " mustard, .62½
20 lb. raisins, .15
3 large tubs, 1.25
6 washboards, .50
50 doz. clothes-pins, .05
10 lb. saleratus, .15

Sales to date, 200.00

Sell C, —
5 boxes sardines, .80

Buy of A, —
7 cd. chestnut wood, 5.75

Sell C, —
3½ lb. Y. H. tea, .70

Sell A, —
20 yd. delaine, .23

Sales to date, 129.76

Buy of A, —
10 pairs kid boots, 3.25
6 " brogans, 2.25

Buy of C, —
1½ tons English hay, 22.50

Sell A, —
5 boxes sardines, .80
50 lbs. B. H. sugar, .09½

C's acct. is settled with cash. 10 per cent off.

Sales to date, 127.83

Mdse. on hand, 2000.00

Items of transactions for day book and ledger:

Cash on hand,	$300.00
Mdse. on hand,	800.00
Horace Hood owes me,	83.75
I owe Geo. G. Gates,	13.50
Buy of Geo. G. Gates, —	
3 Lombardy poplars,	2.15
Buy, —	
6 purple beech,	.75
7 lindens,	1.38
4 golden elms,	.75
5 moss rose,	.50
Sell Horace Hood, —	
100 lb. B. H. sugar,	.05
Buy of Geo. G. Gates, —	
2 Swedish junipers,	1.38
6 mountain ash,	.75
Sell Horace Hood, —	
2 bbl. St. Louis flour,	6.88
Buy, —	
8 neckties,	.38
4 doz. collars,	2.62
6 pairs cuffs,	.42
Buy of Isaac Hart, —	
100 lb. Mocha coffee,	.30
Buy of Geo. G. Gates, —	
3 European larch,	1.12
2 Wisconsin willow,	.62
And pay him,	10.00
Sales to date,	22.75
Buy: 200 arbor vitæ,	.02
6 lilacs,	.28
4 deutzia,	.38
Sell Geo. G. Gates, —	
3 lb. Formosa tea,	.75
And buy of him, —	
8 Austrian pine,	.85
2 Canadian yew,	.95
Paid Isaac Hart,	30.00
Buy: 2 Col. spruce,	1.08
6 maples,	.65
Buy of Geo. G. Gates, —	
3 Norway maple,	.90
And sell him, —	
10 gal. N. O. molasses,	.72
Buy of A. M. Eaton, —	
3 ton English hay,	19.75
Sell Horace Hood, —	
4 lb. Rio coffee,	.28
And he works for me 3 da.,	1.75
Buy of Geo. G. Gates, —	
2 Japan maple,	1.15
2 snowball,	.38
1 Polish juniper,	.92
Buy: 2 pairs brogans,	2.37
2 spades,	.88
2 rakes,	.62
1 sickle,	.58
Sales to date,	72.96
Mdse. on hand,	1000.00

Items of transactions for day book and ledger :

Cash on hand, $50.00
Mdse. " 675.00
A owes me, 100.00
Bills receivable, —
Note No. 1, 200.00
I owe B, 125.00

Buy of A, —
2 tons Franklin coal, 7.50
1 cord pine wood, 6.00
1 cask lime, 2.00

Sell C, —
15 lb. B. H. sugar, .08
And buy of him, —
10 doz. eggs, .20

Sell for cash, —
10 lb. rice, .10
3 doz. eggs, .25
2 lb. coffee, .30
2 gal. cider, .20
6 boxes sardines, .40
1 iron rake, .90

Rec'd int. on bills receivable, Note No. 1 for 8 mo. 15 da.

By B, 25.00

Sell A, —
10 gal. N. O. molasses, .70
1 bbl. St. Louis flour, 6.50
10 lb. raisins, .15
And he pays me, 10.00
And works for me 7 da., 1.75

Pay cash for, —
1 clothes wringer, 5.50

Sales to date, 50.00

Sell C, —
3 doz. Florida oranges, .50
½ lb. tea, .70
And have of him, —
2 bu. potatoes, .90
10 gal. vinegar, .20

Sell for cash, —
3 lb. raisins, .15
2 " chocolate, .25
3 " cheese, .20
10 " butter, .30
2 brooms, .50
1 sieve, .75

Sell A, —
2 pair brogans, 2.50

Credit C with 2 days' work, 1.50

Settle A's acct. with cash.

Sell C, —
3 lb. Rio coffee, .25
2 " tea, .70
1 box mustard, .20
And credit him, —
½ cd. pine wood, 8.00
1 bbl. onions, 6.00

Settle B's acct. with cash.

Mdse. on hand, 700.00

Items of transactions for day book and ledger :

Cash on hand,	$80.00
Mdse. "	790.00
A owes me,	15.00
Bills receivable, —	
Note No. 3,	150.00
Bills payable, Note No. 7,	260.00
I owe C,	25.30
Sell B : 2 umbrellas,	1.25
20 yd. French calico,	.15
12 " gingham,	.18
And credit him,	
2 days' labor,	1.65
3 loads loam,	.75
Pay tax bill,	17.50
Sell for cash, —	
25 yd. sheeting,	.10
10 " drilling,	.15
6 papers pins,	.08
2 pairs slippers,	.90
1 pair brogans	1.75
2 boxes collars,	.30
Pay C,	8.00
Sell B : 12 yd. delaine,	.20
2 pairs rubber boots,	3.00
3 doz. handkerchiefs,	1.75
6 boxes blacking,	.12
And he pays me,	5.00
And brings me, —	
½ ton Eng. hay,	22.50
Pay on Note No. 7, int. for 3 mo. 15 da.	
Buy of C, —	
6 wooden rakes,	.45
1 lawn mower,	11.25
A pays,	8.00
And buys, —	
2 pairs sandals,	.55
1 pair slippers,	.85
Sell for cash, —	
2 spools silk,	.06
10 papers needles,	.08
2 pairs kid gloves,	1.12
1 boot-brush,	.45
Collect int. on Note No. 3 for 7 mo. 20 da.	
A buys, —	
10 skeins yarn,	.25
1 pair blankets,	3.75
And pays,	3.50
Sell for cash : 1 parasol,	1.75
6 pairs hose,	.30
Buy of C : 2 plow-points,	1.10
1 wheelbarrow,	3.50
And pay him,	5.00
Sales to date,	21.73
Pay cash for : 2 shovels,	.95
1 spade,	1.15
Sell B : 3 pairs overalls,	1.15
2 doz. horn buttons,	.12
4 skeins coarse thread,	.09
Mdse. on hand,	820.00

Items of transactions for day book and ledger:

Cash on hand,	$1780.00
Mdse. "	2670.00
A owes me on a/c	700.00
B " "	250.00
I owe C "	1000.00
Buy for cash, —	
1 overcoat,	17.50
10 boxes collars,	.35
Sell C, —	
2 tons Franklin coal,	8.50
And pay him on a/c	100.00
Buy of A, —	
10 bbl. Ohio flour,	8.75
And he pays me on a/c	100.00
And sell him, —	
1½ tons English hay,	25.00
Cash sales to date,	150.72
Buy of B, —	
10 yd. flannel,	.18
30 " French calico,	.17
And he pays me on a/c	100.00
Buy for cash, —	
3 straw hats,	.75
18 cravats,	.20
10 bu. potatoes,	1.15
Cash sales to date,	175.00
Buy of A, —	
6 gal. molasses,	.60
Sell C, —	
2 cd. chestnut wood,	7.75

Rumors of B's unsound financial condition induced me to write to him that I would allow him 30% for an immediate settlement of his account. He brought me at once a note for $50 which he held against Hiram Cook, and cash to balance. (Write the note.)

Sell C: 2 tons hard coal,	6.75
2 bbl. Roman cement,	8.00
1175 ft. lumber,	$20 per M.
Buy for cash: School books,	6.90
2 bottles ink,	.50
2 reams note paper,	1.75
A gives me his note for $200, and $50 in cash.	
Cash sales to date,	160.00
Sell C: 7 cd. pine wood,	5.50
3 bbl. lime,	2.50
1 ton Lehigh coal,	7.25
A works for me 3 days,	2.25
Buy of A: 10 doz. eggs,	.28
50 lb. B. H. sugar,	.09½
5 doz. Messina oranges,	.30
Paid cash for: 6 pr. overalls,	1.75
6 pr. brogans,	2.00
6 shovels,	1.50
Cash sales to date,	90.00
Mdse. on hand,	3000.00

Items of transactions for day book and ledger:

Cash on hand, $200.00
Mdse. " 300.00
Albert Somers owes me, 30.00
Byron Waters " 40.00
I owe Caleb Snow, 15.57

Buy: 100 lb. butter, .24; 40 bbl. apples, 2.60; 10 wooden rakes, .45; 10 bu. oats, .52.

Sell Dennis Rand: 1 bbl. Ohio flour, 6.33; 5 lb. butter, .28.
And he works for me 2 da., 1.62

Sell: 15 lb. butter, profit 25%; 4 bbl. apples, profit 10%; 13 bu. oats, .60; 5 lb. tea, .75.

Albert Somers settles with 10% discount.

Dennis Rand pays 5.00
And buys, —
5 lb. butter, profit 16⅔%.
2 bu. oats, " 8¢.

Sell Caleb Snow, —
20 lb. butter, profit 12½%.
10 bbl. apples, " 25%.
And pay him on ⁸/₀ 8.75

Balance cash.

Buy: 15 lb. veal, .15; 10 yd. flannel, .25; 15 doz. lemons, .22; 12 doz. collars, .25.

Sell: 10 lb. butter, profit 33⅓%.
6 bbl. apples, " 5%.

Sell Caleb Snow, —
25 lb. butter, profit 8⅓%.
And have of him
2 days' labor, 1.75
And cash to balance the butter.

Dennis Rand works for me
3 days, 1.75
And buys, —
2 gal. N. O. molasses, .72

Sell Caleb Snow the balance of the butter at a profit of 20%. Ten per cent of the remaining apples being worthless, he also took the balance of them, profit 20%.

Buy: 12 yd. muslin, .23; 17 yd. drilling, .18; 3 rms. note paper, 1.75; ½ doz. pen-holders, .50; 1 doz. lead-pencils, .40.

Dennis Rand works for me
2 days at 1.88
And buys 10 lb. B. H. sugar, .07

Buy: 6 lb. starch, .08; 3 pairs gloves, .75; 2 inkstands, .42; 1 pair rubber boots, 2.38.

After selling Caleb Snow 1 bbl. St. Louis flour, 7.00; 4 lb. Formosa tea, .75; his account was settled with cash, 10% off.

Mdse. on hand, 400.00

SECTION VI.

Miscellaneous Exercises.

Find if possible the cost of the following without the aid of figures :

1. 18 lb. @ 15¢ ; 24 qt. @ 18¢ ; 42½ lb. @ 8¢ ; 16 cwt. @ 80¢.
2. 34 gal. @ 20¢ ; 26 gal. @ 32¢ ; 8 bbl. at $4.60 ; 68 qt. @ 9¢.
3. 72 yd. @ 15¢ ; 18¼ A. @ $30 ; 17¾ lb. @ 15¢ ; 45 oz. @ 25¢.
4. 18½ doz. @ 35¢ ; 12.2 T. @ $6 ; 28¾ cd. @ $6 ; 152 yd. @ 7¢.
5. 23½ bu. @ 75¢ ; 18¼ gal. @ 42¢ ; 46 qr. @ 14¢ ; 23 da. @ $1.87½.
6. 68 qt. @ 9¢ ; 6¼ cwt. @ $1.30 ; 8½ bbl. @ $5.30 ; 142 oz. @ 8¢.
7. 38 sq. yd. @ 24¢ ; 8 hhd. @ $8.45 ; 18⅜ cd. @ $6.50.
8. 16¼ T. @ $6.50 ; 184 bbl. @ $6 ; 68 qt. @ 17¢ ; 185 oz. @ 9¢.
9. 124 lb. @ 6½¢ ; 45⅗ @ 42¢ ; 18¾ mi. @ $1.50 ; 74 yd. @ 23¢.
10. 87 gr. @ 18¢ ; 28 pwt. @ 16¢ ; 18 cwt. @ $1.40 ; 18 gr. @ $1.20.
11. 48 doz. @ 26¢ ; 28 qr. @ 34¢ ; 27¼ T. @ $4.50 ; 6⅜ bu. @ 75¢.
12. 138 rd. @ 8½¢ ; 7⅔ A. @ $146 ; 18⅜ lb. @ 75¢ ; 54¾ yd. @ 9¢.
13. 50 oz. @ 87½¢ ; 13⅛ bu. @ 72¢ ; 6.5 T. @ $4.75 ; 8.2 yd. @ 17¢.
14. 6 T. 200 lb. of coal at $6 a ton ; at $6.75 a ton.
15. 12 cd. 6 cd. ft. of wood at $8 a cord ; at $6.75 a cord.
16. 28 bu. 3 pk. of wheat at 50¢ a bushel ; at 36¢ a bu.
17. 18 T. 6 cwt. of hay at $15 a ton ; at $1.25 a hundredweight.
18. 68 gal. 3 qt. of syrup at 75¢ a gallon ; at 12¢ a quart.
19. 17 A. 40 sq. rd. at $28 an acre ; at $6 a sq. rd.
20. 68 lb. 8 oz. of butter at 42¢ a pound ; at $35 a hundredweight.
21. 45 bu. 3 pk. of potatoes at 65¢ a bushel ; at 22¢ a peck.
22. 23 qt. 3 gi. of oil at 18¢ a quart ; at 50¢ a gallon.
23. 18 gr. 8 doz. pens at 80¢ a gross ; at 8¢ a dozen.
24. 24 oz. 12 pwt. of silver at 90¢ an ounce ; at 4½¢ a pennyweight.
25. 28 rm. 6 qr. of paper at $1.50 a ream ; at 15¢ a quire.
26. 16 *perches of stone* at $2.40 a perch ; at 10¢ a cubic foot.

1. Write a series of numbers beginning with 3 and increasing by a common difference of 2. What is the 6th term? Write a series beginning with 27 and decreasing by a common difference of 2. What is the 8th term? Each of these series is an *arithmetical series* or a series by *arithmetical progression.* Define arithmetical series.

2. If the 1st term of an arithmetical series is 10, and the common difference is + 4, what must be added to the 1st term to give the 2d term? to give the 4th term? to give the 7th term? If the 1st term is 21, and the common difference is − 3, what is the 4th term? Make a rule for finding any term of an arithmetical series.

3. The 1st term of an arithmetical series is 8, and the common difference is 6. What is the 4th term? What the 9th term?

4. Letting $f =$ 1st term of an arithmetical series, $d =$ common difference, and $n =$ number of terms, find l, or last term of series.

5. Find l of an arithmetical series if $f = 10$, $d = +8$, $n = 6$; if $f = 12$, $d = +3$, $n = 10$; if $f = 30$, $d = -4$, $n = 6$.

6. From the formula $l = f \pm [d \times (n-1)]$ derive the formula for finding d; for finding n.

7. Find d in an arithmetical series of which $f = 8$, $l = 38$, $n = 7$.

8. Find n in an arithmetical series of which $f = 10$, $l = 58$, $d = +8$.

9. If the first term of an arithmetical series is 3, and the common difference is + 2, what is the sum of the first 3 terms? of the first 5 terms? If the series consists of 8 terms, what is the average term? From this, how can you find the sum of the series? Make a rule for finding the sum of an arithmetical series.

10. If f of an arithmetical series $= 8$, $d = 4$, $n = 6$, what is l? What is S, or sum of the series?

11. Give the formula for finding the sum of an arithmetical series.

12. In an arithmetical series, $f = 6$, $d = 8$, $n = 10$. Find S.

13. A man agreed to work 20 days, provided his wages were *increased 5¢* each day. If he received 50¢ for the first day, what *did he receive* for the last day? How much in all?

1. Write a series of numbers which increase by a constant ratio. Write a series of numbers which decrease by a constant ratio. Each of these is a geometrical series. Define geometrical series.

2. Write a geometrical series whose 1st term is 4 and ratio is 2. What is the 3d term? What is the 6th term? To find the 6th term, you multiply the 1st term by what power of the ratio? Make a rule for finding any term of a geometrical series.

3. The first term of a geometrical series is 4, and the ratio is 3. What is the 3d term? What is the 5th term?

4. The 1st term of a geometrical progression is 400, and the ratio is $\frac{1}{2}$. What is the 3d term? What is the 5th term?

5. Letting $f =$ 1st term of a geometrical series, $n =$ number of terms, $r =$ ratio, give the formula for finding l, or last term. From this formula derive the formula for finding f; for finding r.

6. In a geometrical series, $f = 10$, $r = 3$, $n = 6$. Find l.

7. In a geometrical series, $r = 4$, $n = 3$, $l = 972$. Find f.

8. In a geometrical series, $f = 6$, $l = 750$, $n = 4$. Find r.

9. In the geometrical series, f, fr, fr^2, fr^3, S or sum of the series = what? From this equation derive in any way you can the equation $S = \frac{fr^4 - f}{r - 1}$, or $\frac{lr - f}{r - 1}$.

10. In a geometrical series, $f = 6$, $r = 4$, $n = 8$. Find l. Find S.

11. A man agreed to work 10 days on condition that he should have 5 cents the first day, and double the amount on each succeeding day. What did he receive on the tenth day? How much in all?

12. If a child receives \$1 on his 10th birthday, and on each succeeding birthday receives 3 times as much as he received the year before, how much will he receive on his 21st birthday? in all?

13. What will \$100 amount to in 10 years at 6% compounded annually?

14. In how many years will a sum double itself, compounded annually at 8%?

15. By saving 1 cent the first day of the month, and on each succeeding day twice as much as on the previous day, how much would be saved during the month of April?

1. If I mix 3 lb. of sugar worth 8¢ a pound with 5 lb. of sugar worth 6¢ a pound, what is the mixture worth a pound?

2. A mixture of 20 bu. oats @ 35¢, 30 bu. oats @ 32¢, 25 bu. cracked corn @ 50¢ is worth what per bushel?

3. A grocer has two kinds of sugar, which he sells for 6¢ and 9¢ a pound. How many pounds of each may he mix together that the whole may be sold for 8¢ a pound? How many pounds of each may be mixed with 10 lb. of 12-cent sugar that the whole may be sold at 10¢ a pound?

4. In what proportion may tea worth 50¢, 65¢, and $1 a pound be mixed that the whole may be worth 80¢ a pound?

5. How much coffee worth 40¢ a pound must be mixed with 25 lb. of coffee worth 32¢ a pound to make a mixture worth 35¢ a pound?

6. How much sugar worth 6¢, 7¢, and 9¢ must be taken to make a mixture of 100 lb. worth 8¢ a pound?

7. How much water must be mixed with vinegar worth 40¢ a gallon to make 50 gal. worth 30¢ a gallon?

8. A silversmith wishes to make a dish weighing 1 lb. 6 oz. of silver 800 fine. How much pure silver must be used?

9. How much pure gold in 100 sovereigns, if each sovereign weighs 123.374 gr. and $8\frac{1}{3}\%$ of the coin is copper?

10. If 279 ten-mark pieces can be made from 1^{Kg} of pure gold, how much will 1 piece weigh, the alloy being 1 part in 10?

11. How much pure gold in a watch-case that weighs 8 oz. and is 18 carats fine?

12. How much alloy must be put with 6 oz. 10 pwt. of pure gold to make a gold cup 16 carats fine?

13. A silver bracelet 850 fine weighs 3 oz. 4 pwt. What is the cost of the pure silver contained in it at 90¢ an ounce?

14. If 2 gal. of beer containing 3% alcohol, and 4 gal. of beer containing 4% alcohol be mixed with 3 gal. of water, what per cent of alcohol is the mixture?

15. *How much* pure silver must be used to make a plate weighing *2 lb. 4 oz., 840 fine?* How much to make 50 plates, each 1 lb. 2 oz.?

1. A man in business purchases \$3000 worth of goods each month for 10 months, and \$5000 worth the 2 remaining months of the year, and gets 3% off for cash, or three months' time. He has only half enough money to pay cash for his purchases. He borrows of the bank the other half on 3 months' time, paying 6% interest, and thus is enabled to take advantage of the 3% for cash on all his bills. What do the goods cost?

2. A merchant sells \$300 worth of goods a month, and makes 20% profit. He pays his salesman 10% for selling, and his expenses amount to $16\frac{2}{3}$% of the amount of sales. What is his loss or gain for the year?

3. Write the following kinds of business paper, and make problems which call for their use: bill of sale; order; due-bill; receipt; promissory note bearing interest; promissory note on demand; joint and several note; time note without interest; sight draft; time draft.

4. What is the interest of \$4120 from Aug. 26, '90, to May 27, '92, at 5%?

5. What is the interest of \$6987.66 from Nov. 5, '90, to May 27, '92, at 5%?

6. What is the interest of \$2907 from Dec. 2, '90, to May 27, '92, at 5%?

7. A customer hands you in the paper business a sheet of paper that measures $32\frac{1}{2} \times 46$ inches, and weighs 140 lb. to the ream of 480 sheets. He wants made a paper of the same thickness that will measure 24×38 inches and contain the same number of sheets to the ream. What will it weigh to the ream? What would it weigh if it were made 500 sheets to the ream?

8. Average the following account: Due Jan. 1, \$150; Jan. 10, \$50; Jan. 21, \$40; and Feb. 15, \$70.

9. A man takes out an endowment policy yielding in 20 years \$8000, upon which a premium of \$250 a year is to be paid. What rate of interest does he get?

10. Which is the better investment, — stock at 125, paying $7\frac{1}{2}$% dividend, or stock at 80, paying 5%?

1. What is the balance Dec. 11, 1892, of a demand note for $400 with interest at 6%, dated Jan. 3, 1892, on which a payment of $300 was made Aug. 14, 1892?

2. A ninety-day note for $175, dated Jan. 5, was discounted Jan. 20. Find proceeds, discount 6%.

3. A note for $400 at 6 mo., dated Jan. 23, 1892, was discounted March 29, 1892. Find proceeds.

4. $1796 was paid at auction for 16 shares of bank stock. What was the price per share? The price paid for this stock per share being $\frac{1}{4}$ of 1 per cent less than 50% premium, what is the par value of the shares? What would this stock net the investor at the price paid, the regular dividends being 8% per annum on the par value?

5. A demand note of $1555, dated Nov. 17, 1892, at 6% interest, has had the following payments made: Dec. 1, $555; Dec. 10, $332; Dec. 30, $200; Jan. 10, $468. How much interest was due Jan. 10, when the note was paid in full?

6. A demand note for $4512, dated Nov. 22, 1892, at 7%, has paid on it Dec. 5, $721; Dec. 9, $758; Dec. 23, $758; Dec. 27, $1516; Jan. 10, $759. How much interest is due Jan. 10?

7. A waterworks company wishes to build a wall 1 ft. thick and 9.4 ft. high in circular shape so as to inclose a space 42 ft. 8 in. in diameter. How many bricks will it take, reckoning 21 bricks to the cubic foot?

8. If a steel tape is correct at 68° F., and lengthens one part in sixty-five million (.000065) for each degree of rise in temperature, what is the true length of a distance which it indicates to be 5280 ft. long when the thermometer reads 110°? What when the thermometer reads 20° below zero?

9. How much would a manufacturer receive for 10 kegs of 4^{d} nails at $2.40 per keg, freight allowance of 20¢ per keg, and cash discount of 2%?

10. If 1 screw is made from $\frac{1}{4}''$ wire $1\frac{1}{4}''$ long, what will the wire *cost at* $3\frac{1}{2}$¢ per pound, sufficient to make 1 gross screws, allowing *35% for waste?*

1. If one bolt weighs .4 of an ounce, how much will 50 lb. of bolts cost at 80¢ per hundred, at 60, 10, and 5 off?

2. There are three times as many rivets as burrs in packages. How much in weight of each is there in a half-pound package?

3. A commission merchant sells 180 bbl. of flour at $5.25 a barrel, and 850 bu. of wheat at 80¢ a bushel. With the proceeds, after deducting his commission of 2½% for selling and 1½% for buying, he buys cloth at $2.40 a yard. How many yards of cloth does he buy?

4. Bought a cargo of flour containing 4650 bbl. for $20,000. 18% of it was thrown overboard in a storm; the balance was sold at the rate of $5.20 a barrel. What was the gain or loss?

5. A man, after drawing out 20% of the money he had in a bank, and then 10% of the remainder, found that he had $612 left. How much had he in the bank at first?

6. If 25% is gained by selling cloth at $2.75 a yard, what would be gained or lost by selling it at $2.37½ a yard? at $2.12½?

7. A room is 18 ft. by 16½ ft., and 10½ ft. high, with 4 windows, each 5½ ft. by 4½ ft., and 2 doors, each 7½ by 5 ft. If laths are 40¢ a bundle, and cost of laying is 5¢ a square yard, what will it cost to lath the walls and ceiling, ½ allowance for openings? How many rolls of paper are needed for the walls? How many yards of carpet 27 in. wide for the floor, the carpet being laid breadthwise, and no loss allowed for matching?

8. In a report* of the United States Commissioner of Labor, the following facts were given: The average annual expense of 928 American families whose head was engaged in the iron or coal industries was as follows: Food, $206.37; rent, $70.87; fuel, $22.43; lighting, $4.57; clothing, $82.07; sundries, $103.13. Of 338 European families in the same circumstances, the average annual expense was as follows: Food, $191.99; rent, $42.24; fuel, $18.40; lighting, $5.33; clothing, $60.35; sundries, $63.57. Make as many problems as you can from these figures.

* Report of 1890.

1. A man wishes to build a barn 94 ft. long, 40 ft. wide, and 30 ft. high to the eaves. The gable is to be 15 ft. high. (*a*) How many feet of lumber will it take to board the sides and ends with $\frac{3}{4}$ in. boards, allowing for two double doors 15 ft. by 12 ft., and 2 single doors 8 ft. by 5 ft.? (*b*) How many feet of lumber will be required for the floor, it being laid with 2-in. plank? (*c*) How many feet of boards will be required for the roof? (*d*) How many bundles of shingles laid $4\frac{1}{2}$ in. to the weather will be required for the roof? (*e*) How many bundles of shingles laid 4 in. to the weather will be required for the gable ends? (*f*) How many feet of lumber required to make the doors, the boards being $1\frac{1}{2}$ in. thick? (*g*) Find the cost of the lumber required, the boards and plank being \$15 per M., and the shingles being \$4.25 per M.

2. The sales of a grain-dealer in one year amounted to \$100,000. $\frac{2}{5}$ of the receipts were for rye, on which he made $12\frac{1}{2}\%$ profit; $\frac{2}{5}$ were for wheat, on which he made 20% profit; and the remainder for other grains, on which 5% profit was made. What was the cost of all the grain he sold?

3. At the end of a year a firm found its profits to be \$27,000, which was $12\frac{1}{2}\%$ of their capital. A had put $33\frac{1}{3}\%$ of the capital into the business, B $37\frac{1}{2}\%$, and C the remainder. What was each man's share of the profits?

4. If \$360.75 is $16\frac{2}{3}\%$ of my money, and $6\frac{1}{4}\%$ of mine is equal to 10% of my sister's, how much more money have I than my sister?

5. How many bushels of shelled corn weigh as much as 100 bu. of corn on the cob? How many bushels of corn meal?

6. By a scale of $\frac{1}{8}$ of an inch to a rod, draw the plan of a lot of land described as follows: From a point *A* to a point *B* east is 10 rods; from *B* to a point *C* south is 4 rods; from *C* to a point *D* east is 8 rods; from *D* to a point *E* south is 12 rods; from *E* to a point *F* west is 18 rods; join *AF*. (*a*) Find the number of yards of fence that will be required to inclose the lot. (*b*) Find the square contents in acres, etc. (*c*) Join *BF* in the plan, and find the area of *ABF*. (*d*) Join *DF* in the plan, and find the area of *DEF*. (*e*) Find the area of *BCDF*.

1. If oak wood is .85 as heavy as water, and poplar is .38 as heavy as water, how much poplar will weigh as much as a cord of oak? How much oak wood will weigh as much as a cord of poplar?

2. The difference of time between New York and Geneva is 5 hr. 20.5 min., and between New York and the City of Mexico is 1 h. 40.5 min. What is the difference of longitude between Geneva and the City of Mexico?

3. Find on the map two places whose latitude is about 40° N. lat. How many miles between the two places?

Make all the problems you can from the following:

4. Coal sells at \$6.50 a ton. A man burns an average of 1800 lb. a month from Oct. 1 to May 15.

5. The list price of nails is \$12.40 per cwt., 10 and 5 off.

6. Flour is worth \$5.80, due in 60 da., 5 off for cash.

7. A cart is 9 ft. long, $4\frac{1}{2}$ ft. wide. The wood is 4 ft. long. The horses can draw a load of 6 cd. ft. Wood is worth \$6.50 a cord.

8. A house is insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}\%$ for 5 years. It burns down.

9. A farm is sold for \$2800. The buyer has but \$1200 in cash.

10. There is a field of $18\frac{1}{2}$ A. Wheat is raised on it, 45 bu. to the acre. The farmer's bin is $18\frac{1}{2}$ ft. long, 6 ft. wide. The market price of the wheat is 68¢ a bushel cash, 70¢ in 60 days.

11. How many $4'' \times 6\frac{1}{2}''$ pass-books of 36 pages each can be made from $1\frac{1}{2}$ reams of paper, the sheets being $23'' \times 26''$?

12. If Mercury is $\frac{1}{15}$ the size of the earth, and the moon is $\frac{1}{81}$ the size of the earth, how do Mercury and the moon compare in size?

13. Construct on the blackboard a rectangle, a parallelogram, a triangle, and then find the area of each by the aid of a formula.

14. The three dimensions (inside) of a box are each equal to 1 ft. 3 in. How many square inches of cloth will it take to line it? If the boards of the box are $\frac{3}{4}$ in. thick, how many square inches of cloth will it take to entirely cover the outside?

15. The area of a square field is $\frac{1}{2}$ A. What is the length of one *side in rods? in* rods and feet?

1. The perimeter of a rectangle is 148 rd. 8 ft., the width is 8 rd. 2 ft. Find the length and area.

2. I wish to build a brick house 48 ft. long and 32 ft. wide. At $8 per thousand, how much will the bricks cost for the four walls 22 ft. high, allowing for 12 windows 6 ft. × 4½ ft., and 3 doors 7½ ft. × 5 ft., the walls to be 12 in. thick, and the bricks to be 8 in. × 4 in. × 2 in. ?

3. How many perches of stone will be required for the cellar of the above house 7 ft. deep?

4. How many bricks will be required to build a 40-foot chimney having walls 4 in. thick and a flue 8½ in. by 5 in. ?

5. The following table shows the value of the imports and exports of the United States in American and in foreign vessels during each fifth year since 1856. Find in each case the per cent (three places) carried in American vessels.

Year ending June 30.	In American Vessels.	In Foreign Vessels.	Per Cent in American Vessels.
1856	$482,268,274	$159,336,576	.....
1861	381,516,788	203,478,278	.....
1866	325,711,861	685,226,691	.....
1871	353,664,172	755,822,576	.....
1876	311,076,171	813,354,987	.....
1881	250,586,470	1,269,002,983	.....
1886	197,349,503	1,073,911,113	.....
1891	206,439,925	1,450,101,087	.....

6. Required the number of strips of wall paper for the walls of a room 18½′ long, 12′ wide, and 9½′ high. There are 3 windows, each 5½′ by 4′, and 2 doors, each 7′ by 4½′. How many rolls will be needed? Cost of border at 12¢ a yard? How many yards of carpet 27 in. wide will be needed for the floor?

7. *How many* barrels of water have fallen upon an acre of land, *if the rainfall* has been ¾ in. ?

1. If the rate of taxation is 14 mills on a dollar, and I pay $142.10 tax, for what is my property valued?

2. If the diagonal of the floor of a square room is 22 ft. 6 in., what is its area?

3. A collector receives $3.60 for collecting a bill, which is 3% of the face of the bill. What is the face of the bill?

4. If I melt a lead ball 4 in. in diameter, how many bullets $\frac{1}{4}$ in. in diameter can I make from the lead?

5. A and B received the contract to do a certain piece of work for $1200. A furnished 12 men 18 days and 6 men 14 days, and B furnished 15 men 20 days and 6 men 17 days. What was the share of each contractor?

6. How much cloth 30 in. wide will cover 12 tables 5 ft. by $2\frac{1}{2}$ ft.?

7. A owes B $800, payable July 1. If he should pay $300 of it June 5, when should the balance be paid?

8. In a store the "key" by which goods were marked was "panegyrist," the letters denoting the cost being above a line and those denoting the selling price below; thus, cloth marked "$\frac{p}{a}$" as cost would be marked "$\frac{p}{g}$" as selling price if the gain was 25%. Using this key as a guide, supply the blanks in the following, the gain in each being 20%:

$$\frac{pg}{\quad};\ \frac{\quad}{an};\ \frac{pag}{\quad};\ \frac{rt}{\quad};\ \frac{\quad}{net}.$$

9. Using the key denoted in the last problem, give the gain or loss per cent of goods marked thus:

$$\frac{eg}{gt};\ \frac{st}{ip};\ \frac{gtt}{eyt};\ \frac{ngy}{ept};\ \frac{gy}{it}.$$

10. A cistern is formed like the frustum of a cone, having bases $4\frac{1}{2}$ ft. and 6 ft. in diameter and 8 ft. deep. How many barrels of water will it hold?

11. A cask whose head diameter is 1 ft. 6 in., bung diameter 1 ft. 11 in., and length 3 ft., contains how many gallons of vinegar?

ASSESSED VALUATION OF REAL AND PERSONAL PROPERTY IN THE UNITED STATES. FROM THE CENSUS REPORT OF 1890.

States and Territories.	Total Assessed Valuation.		States and Territories.	Total Assessed Valuation.	
	1880.	1890.		1880.	1890.
Alabama	$122,867,228	$197,080,441	Montana	$18,609,802	$106,392,892
Arizona	9,270,214	21,434,767	Nebraska	90,585,782	184,770,305
Arkansas	86,409,364	172,408,497	Nevada	29,291,459	24,663,385
California	584,578,036	1,071,102,327	N. Hampshire	164,755,181	252,722,016
Colorado	74,471,693	188,911,325	New Jersey	572,518,361	688,309,187
Connecticut	327,177,385	358,913,906	New Mexico	11,363,406	46,041,010
Delaware	59,951,643	74,134,401	New York	2,651,940,006	3,775,325,938
Dist. Columbia	99,401,787	153,307,541	North Carolina	156,100,202	212,697,287
Florida	30,938,309	76,926,938	North Dakota	8,786,572	78,394,536
Georgia	239,472,599	377,366,784	Ohio	1,534,360,508	1,778,138,457
Idaho	6,440,876	25,581,305	Oregon	52,522,084	166,025,731
Illinois	786,616,394	727,416,252	Pennsylvania	1,683,459,016	2,592,841,032
Indiana	727,815,131	782,872,126	Rhode Island	252,536,673	321,764,503
Iowa	398,671,251	478,318,248	South Carolina	133,560,135	132,182,638
Kansas	160,891,689	290,593,711	South Dakota	11,534,958	131,592,587
Kentucky	350,563,971	512,615,506	Tennessee	211,778,538	347,510,103
Louisiana	160,162,439	234,320,780	Texas	320,364,515	695,842,320
Maine	235,978,716	309,129,101	Utah	24,775,279	104,758,750
Maryland	497,307,675	482,184,824	Vermont	86,806,775	161,551,328
Massachusetts	1,584,756,802	2,154,134,626	Virginia	308,456,135	362,422,741
Michigan	517,666,359	945,450,000	Washington	23,810,693	124,795,449
Minnesota	258,028,687	588,531,743	West Virginia	139,622,705	169,927,587
Mississippi	110,628,129	157,518,906	Wisconsin	438,971,751	592,890,719
Missouri	532,795,801	786,343,753	Wyoming	13,621,829	31,431,495

1. What per cent of gain in valuation during the period from 1880 to 1890 in your State?

2. In which States was the gain per cent less than in yours during the same time? In which was it more?

3. What per cent of the valuation of New York in 1890 was the valuation of Massachusetts? of California? of Pennsylvania?

4. What State made the greatest absolute gain in valuation? How much was the gain?

5. What State made the greatest relative gain in valuation? What per cent was the gain?

6. Find the total assessed valuation for all the States and Territories for 1880; for 1890.

7. Find, if you can, the assessed valuation *per capita* in your State in 1890.*

8. At the legal rate of interest, what is the interest of the valuation *of your State* per day? (See Appendix.)

* See page 20, Book No. IV, for census table.

1. Estimating the year as $365\frac{1}{4}$ days, how much time is gained in a century?

2. How many acres of land are covered by the Capitol at Washington, which is 751 ft. long and 348 ft. wide?

3. How many English miles is a German mile, which measures 8106 yards?

4. It is by one route 3540 nautical miles from New York to Liverpool. How many common miles between the two places?

5. A bin $24' \times 6\frac{1}{2}' \times 4\frac{1}{4}'$ contains how many bushels of wheat? How many bushels of apples? beets? carrots? potatoes? corn on cob? salt? (See Appendix.)

6. How many perches of stone in a wall 20 rd. long, 4 ft. high, and 18 in. thick?

7. How many cords of stone per acre in a quarry, if the average depth of stone is 20 inches? How many tons, specific gravity being 2.72?

8. How many bushels of each of the following commodities will it take to weigh a ton: wheat? rye? oats? barley? salt? potatoes? coal?

9. How many bushels of rye will it take to weigh as much as 100 bushels of wheat?

10. How much does 3 qt. and 1 pt. of shelled corn weigh?

11. If wheat sells for 80¢ a bushel, how much would have to be paid for a ton at the same rate?

12. I bought 2 T. of potatoes for $50, and sold them at the rate of 90¢ a bushel. How much did I gain or lose?

13. How much more does 100 bu. of corn weigh than the same quantity of corn meal?

14. I buy a cargo of oats for $340. The weight was 10 T. 6 cwt., and sold them for 60¢ a bushel. What was the profit?

15. A wheel whose radius is 60 in. has 32 teeth. How large a wheel will be needed to have 64 teeth of the same size?

16. There is needed in a machine a wheel to revolve once while another wheel revolves five times. If the latter wheel is 12 in. in *diameter, what* must be the diameter of the former?

1. Fill out blanks in the following table (for 1890):

Countries.	Population.	Children Enrolled in Schools.	Ratio to Population.
			Per Cent.
Bavaria	5,589,382	1,187,792	.....
Belgium	6,147,041	.....	13.5
Sweden	.....	736,790	15.4
Netherlands	4,564,565	657,611	.....
United States	.....	14,377,536	23.3

2. In London a certain company builds blocks of tenements for 3*d.* a cubic foot. How much will it cost to build a block 50 ft. by 100 ft. on ground, and 50 ft. from cellar to bottom of cornice, the roof having an angle of 90° at the ridge-pole?

3. A car-load of wheat weighing 16½ T. is shipped from Illinois. What is the cost at 85¢ a bushel?

4. Estimate the number of square feet of ¾″ box-boards that can be piled into a box-car 16 ft. long, 8½ ft. wide, and 6 ft. high, the boards being 48 in. long.

5. A trader bought 400 tons of coal at $4.75 a ton (long ton), paid 75¢ a ton for freighting, and sold it at $6 a ton (short ton). What per cent did he gain?

6. December 1, 1866, the principal of the public debt of the United States was $2,773,236,173.69, and twenty-five years later it was $1,546,961,695.61. Required the percentage of decrease.

7. I own a farm mortgaged for ⅔ of what it cost me. I sell it for $6000, which is 25% more than it cost me. After paying off the mortgage, how much money is left from the proceeds of the sale?

8. A clock which loses 2 minutes a day is set to the correct time at 9 A.M. on Tuesday. What will be the correct time when the clock marks four o'clock on the following Friday afternoon?

9. If I sell ⅔ of my goods at 25% profit, at what rate of loss *must I sell the* remainder to neither gain nor lose on the whole? *At what rate* to gain 15% on the whole?

1. A merchant failing in business has assets as follows: real estate, $4680; furniture, $1240; cash, $450. Accounts valued at $1380. Mr. Brown holds a mortgage of $1500 on the real estate. He owes Mr. Smith $1200, and all other creditors $11,840. The assignee receives 5% on the assets, and other expenses amount to $460. What sum will Mr. Smith receive from the estate?

2. A jobber is offered by one manufacturer 40% discount from the list price, and by another 33⅓% and 10% off. If the list price is the same in both cases, which is the better offer? Which is the greater discount, — 30 and 10 and 5, or 40 and 5, and how much would be saved in buying $800 worth of goods of the cheaper party?

3. A merchant paid $3250 for wool, and sold it at 18% above cost, taking payment in cotton cloth, which he sold at a loss of 15%. Did he gain or lose, and how much?

4. I sold my horse and carriage, gaining $150 or 25% of their cost. The horse cost 62½% less than the carriage. What was the cost of each?

5. A cylindrical tank is 4 ft. 6 in. high and 18 in. in diameter. How many gallons will it hold?

6. A vessel that sails 12 knots an hour would sail at the same rate how many common miles in 5 days?

7. The operatives of a certain factory who work 9½ hours a day refuse to submit to a reduction in wages of 10%, but are willing to submit to a corresponding extension of time, provided they can have the same daily wages. If this plan is carried out, how many hours a day will the operatives be required to work?

8. What is the amount of $1200 for 12 yr. 8 mo., interest compounded annually at 4½%? (See Appendix.) How much if the interest is 3½%? 6%?

9. Find the number of board feet in 4 planks, each 2½ in. thick, 9½ ft. long, 12 in. wide at one end and 9 in. wide at the other.

10. At what premium should I purchase a 10% $100 bond (interest payable semi-annually) having 4 years to run, so that my money invested will yield me 4% during that time?

1. Up to January 1, 1893, the fastest time in making a mile by a trotting horse was 2 min. 4 sec.; by a bicycle, 1 min. 56$\frac{3}{4}$ sec.; by a running horse, 1 min. 35$\frac{1}{2}$ sec.; by a railroad train 37 sec. If they were following each other in that order 12 miles apart and went at the given rate, in how many minutes would the railroad train pass each of the others? In how many minutes would the running horse catch up to each of those ahead? In how many minutes would the trotting horse find himself behind all the rest?

2. Find the contents in bushels of a bin 6′ 8″ × 4′ × 3′ 8″; of a bin 9′ 8″ × 6′ 3″ × 2′ 9″; of a bin 12′ 2″ × 9′ × 3′ 6″.

3. How many bushels of wheat can be put into a car 32 ft. 6 in. long, 7 ft. 8 in. wide, and 6 ft. 6 in. high?

4. How many gallons can be put into a rectangular cistern 8 ft. 6 in. by 4 ft. and 3 ft. 6 in. deep?

5. A tank in the form of a cylinder is 8 ft. high and 6 ft. in diameter. How many gallons will it hold? How deep will the water be to measure 20 barrels of 31$\frac{1}{2}$ gal. each?

6. How many barrels of water will be required to fill a tank 8 ft. square on the bottom and 6 ft. high?

7. Write a demand note bearing interest at the legal rate, and having two or more indorsements. State relations between the parties.

8. Write a note such as might be discounted at the bank at the legal rate. State relations between the parties.

9. A piece of tinfoil is 24.6^{cm} long and 14.5^{cm} wide; it weighs 5.2^{g}. If the tin is 7.29 times as heavy as an equal bulk of water, how thick is the foil?

10. The diameter of a circular vessel is 4.4^{cm}; its depth is 10.5^{cm}. Calculate its contents in cubic centimeters. The measured contents are 158$^{cu\ cm}$. Find the difference between the measured and the computed distance; also what per cent this difference is of the measured contents.

11. Make problems about the weight of cart-loads or barrels of *commodities* named in the table of specific gravities, page 116, the *cart or barrel* being of given dimensions.

1. The breaking strength of wires — that is, the force required to pull them apart — should be proportional to the square of their diameters. One wire whose diameter was .412^{mm} broke with a weight of 7.08 lb. What should be the breaking strength of another whose diameter was .546^{mm}? The actual weight required was 12.92 lb. Find the difference between the actual and the computed results; also what per cent this difference is of the computed result.

2. A meter of wire weighs 2.66^{g}; its breaking strength is 8.1^{Kg}. How long must a piece of the wire be which, when suspended, will break by its own weight?

3. The times of vibration of different pendulums are as the square roots of their lengths. A pendulum 39.1 in. long swings once in a second. A meter equals 39.37 in. How many times per minute will a pendulum a meter long swing?

4. A pendulum is observed to swing 50 times in 55 sec. How long is it? How long is the pendulum of a clock which ticks twice per second?

5. An elevator well is 130 ft. deep. How long would it take the longest pendulum which could be hung in the well to swing?

6. A falling body falls 16.08 ft. in the first second, 48.24 ft. in the second, 80.40 ft. in the third second, or in each second of its fall 16.08 ft. multiplied by one less than twice the number of the second. A stone dropped from a balloon falls 15 seconds. How far will it fall in the 5th second? in the 10th? in the 15th?

7. The distance that a body falls in 1 second is 16.08 ft., in 2 seconds is 64.32 ft.; or, in general, 16.08 ft. multiplied by the square of the number of seconds that the body is falling. How high up was the balloon in the preceding question?

8. The velocity of the falling body at the end of the first second is 32.16 ft. per second, at the end of the second second 64.32 ft. per second; or, in general, 16.08 ft. multiplied by twice the number of the second. If a ball is dropped from the top of the Washington Monument, 555 ft. high, how fast will it be going when it reaches the ground?

1. Two weights, or forces, acting in the same direction on opposite sides of the point of support of the beam to which they are applied will balance each other, when the product of the number of units of force by the number of units of distance of the force from the point of support is the same on both sides. A boy who weighs 80 lb. plays see-saw with another who weighs 100 lb. If the first sits five feet from the point of support, how far away from the point of support must the other sit?

2. A kilogram equals very nearly 2.2 lb. With a kilogram weight at one end of a yard-stick and a pound weight at the other end, where must the stick be supported to balance, disregarding the weight of the stick?

3. The principle given above applies when there are several weights used instead of one on each side. A yard-stick is supported at its middle point. A weight of 2 lb. is hung on one end, a weight of 5 lb. on the other, a weight of 3 lb. half-way between the 2 lb. weight and the middle point. Where shall 4 lb. be hung that the stick may be horizontal?

4. A long stick is balanced on a support. A weight of 100^{g} is placed 31^{cm} from the point of support, and the support moved until the stick is again balanced. The point of support is now 24^{cm} from where it was at first. Assuming that the stick acts as it would if all its weight were at the first point of support, how much does the stick weigh?

5. When a beam carrying a weight rests upon two points of support, the weight on each point of support is inversely proportional to the distance of the weight from that point. A yard-stick hangs by the ends from two spring balances; a weight of 12 lb. is hung 1 ft. from one end. What is the reading of each balance?

6. A weight of 9 lb. is hung 1 ft. from the other end. With both weights on, what is the reading of each balance?

7. A man and a boy are poling hay on poles 10 ft. long. If one pile of hay weighs 60 lb. and another 40 lb., how far from each end of the poles shall each pile be that the man may carry 60% of the *load? Is more* than one answer possible? If so, find another.

1. With the ordinary arrangement of pulleys, disregarding friction, the weight supported equals the power multiplied by the number of pulleys used. With two pulleys in each block, how much force will be necessary to raise a barrel of flour?

2. With three pulleys in the upper block and two in the lower, how much force would you have to use to raise yourself?

3. With a wheel and axle, disregarding friction, the ratio of the power to the weight equals the ratio of the radius of the axle to the radius of the wheel. How much force will be required to raise a man who weighs 150 lb. from a well, if the diameter of the axle is 8 in. and the handle of the windlass is 2 ft. long?

4. With three pulleys in each block, if the rope is wound on an axle 3 in. in diameter, turned by a handle 2 ft. long, how much force will be required to raise a rock which weighs 1920 lb.?

5. With an inclined plane, if one pushes parallel to the incline, the ratio of the power to the weight equals the ratio of the length of the plane to its height. With a pair of skids 8 ft. long, how much force will it take to roll a barrel of flour into a cart 3 ft. high?

6. The pressure on the sides of a vessel containing a liquid equals the weight of a mass of the liquid the area of whose base equals the area of the lateral surface of the vessel, and whose height equals the average depth of the liquid, or the depth of the middle point. A standpipe 80 ft. high and 30 ft. in diameter is full of water. What is the total pressure on the sides, if one cubic foot of water weighs $62\frac{1}{2}$ lb.? If the standpipe is made of plates 5 ft. wide, what is the pressure on the lowest tier of plates?

7. A floating body displaces its own weight of the liquid in which it floats. A body which sinks in a liquid loses an amount of weight equal to the weight of its own volume of the liquid. The ratio of the weight of any body to the weight of an equal volume of water is called the specific gravity of the body. If the specific gravity of a body is .6, how much of the body will be above the surface when it floats in water? How much when it floats in a solution of salt, whose specific gravity is 1.1? How much when in alcohol, specific gravity .8?

1. A rock whose specific gravity is 2.5 and which weighs 100 lb. in air, will weigh how much in water?

2. How much will the same rock weigh in sea-water, specific gravity 1.027?

3. The specific gravity of ice is .93. If a floating block is 1 ft. thick, what will be the thickness of the portion above water?

4. The specific gravity of seasoned spruce is .512. A cubic foot of water weighs 62.5 lb. A raft is made, two planks thick, of twenty 2-inch planks, each 12 ft. long and 1 ft. wide. How many boys whose average weight is 100 lb. will the raft support? How many would it support if it were one plank thick, the same planks being used as before?

5. The specific gravity of boxwood is 1.28, that of maple is .675. A block which will just float in water was made of these two kinds of wood. What part of its thickness is of each kind?

6. A piece of cast iron weighs in air 3 lb. 3 oz.; in water it weighs $2\frac{3}{4}$ lb. What is its specific gravity?

7. A body weighs 5 lb. in air; it weighs 3 lb. in water; in a solution of a salt it weighs $2\frac{3}{4}$ lb. What is the sp. gr. of solution?

8. A piece of glass 5^{cm} square weighs in air 13.3^{g}; in water, 8.1^{g}. What is its specific gravity? How thick is it?

9. An empty bottle weighs 83.6^{g}; full of water it weighs 163.8^{g}; the same bottle full of a solution of copper sulphate weighs 174.4^{g}. What is the specific gravity of the solution? When the same bottle is filled with a solution of sodium nitrate it weighs 166.9^{g}. What is the specific gravity of this solution?

10. A piece of copper weighs 30.5675^{g}; some of the copper is dissolved by putting on it a few drops of nitric acid; the dissolved copper is washed off in a liter of water; the piece of copper now weighs 30.5274^{g}. How much copper is there in each cubic millimeter of the water?

11. When the mercury in the barometer stands at 30 in., water can be raised about 33 ft. in a common pump. How high could *water be raised* by such a pump on Mount Washington when the *barometric reading* was 608^{mm}?

1. The volume of an inclosed gas should vary inversely with the pressure applied. In an experiment, under a pressure of 805^{mm} of mercury, the length of the column of gas used was 26.45^{cm}. What should it have been under a pressure of 1121^{mm}? As measured it was found to be 19^{cm}. Find the difference between the actual and the computed results; also what per cent this difference is of the computed result.

2. A block is placed on a smooth board; one end of the board is raised until the block begins to slide down the board; the raised end is 26^{cm} above the table; the distance from a point on the table directly under the raised end of the board to the other end is 46^{cm}. Find the ratio of the first distance to the second. It is called the coëfficient of friction of the block on the board.

3. When the block is used on a pane of glass, the measurements are 18.4^{cm} and 49^{cm}. Find the coëfficient of friction in this case.

4. When a piece of glass is glued on the lower surface of the block, and the block slides on the pane of glass, the measurements are 15.3^{cm} and 49.9^{cm}. Find the coëfficient in this case. Try these three experiments, and see what results you get.

5. A piano wire, or any other instrument which gives middle C, vibrates about 264 times per second; the ratio of the numbers of vibrations for each tone of the scale after the first to the number for the first one is $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{15}{8}$, 2, taking the tones in order. Find the vibration numbers for the other tones when middle C is 1.

6. The amount of heat required to melt a pound of ice will warm 1 lb. of water about 144°, or 144 lb. 1°, or will be given off by similar amounts in cooling through similar ranges. A piece of ice which weighs 5 lb. is put into 2 gal. of water. If a gallon of water weighs about 8 lb. 6 oz., how much should the water be cooled in melting the ice?

7. Light travels about 186,000 miles a second. The light from the nearest fixed star is about 3 yr. 6 mo. in coming to the earth. How far away is the star?

8. How many miles away is a cannon if the report is heard just *one minute* after the flash is seen?

1. It takes about .43 as much heat to warm any amount of spirits of turpentine as it does to warm an equal amount of water through the same range of temperature. If ten pounds of water are warmed from 70° to 120°, how much will the temperature of an equal weight of turpentine be raised under like conditions?

2. The brightness with which any surface is illuminated by a given source of light varies inversely with the square of the distance from that source. One person holds his book $1\frac{1}{2}$ ft. from a 12 candle-power lamp. How far must another hold his book from a 16 candle-power lamp that the page may be as brightly lighted as in the first case?

3. The resistance of 100 yd. of No. 25 copper wire is 7.86 ohms; resistance varies inversely with the square of the diameter of the wire; the diameter of No. 25 wire is .02 in.; the diameter of No. 31 wire is .01 in. What would be the resistance of 10 yd. of No. 31 wire?

4. Resistance varies inversely with the conducting power. The conducting power of copper is 96, that of German silver 7.5. What would be the resistance of pieces of German silver wire of like dimensions in each part of preceding question?

5. The weight of bodies below the earth's surface decreases as distance from the centre decreases. How much will a body weigh 2000 miles below the surface? 3000 miles from the centre of the earth? at the centre?

6. A body weighing at the surface 100^{Kg} is carried below the earth's surface till it weighs only 70^{Kg}. Where is it?

7. As bodies rise above the earth's surface they decrease in weight according to the square of the increase in distance from the earth's centre. How much does a 100 lb. ball weigh 4000 miles above the earth (*i.e.* 8000 miles from the centre)? 12,000 miles from the centre? 16,000 miles from the surface? 8000 miles from the surface? 12,000 miles from the surface?

8. How far away from the earth's surface must a 49 lb. rock be *carried to weigh* only 1 lb.? (Notice that in the above law the *distance is reckoned* from the earth's centre.)

1. At one point of the Grand Cañon of the Colorado the walls rise 7000 ft. above the river. If a boulder weighing several tons were rolled over the edge of the precipice at the top, how long would it be before the crash of the stone on the rocks below would be heard by a person at the top?

2. The intensity or loudness of a sound varies inversely as the square of the distance. If A is 50 rd. from a bell, and B is 70 rd. from it, how will the loudness of the sound as heard by B compare with the loudness as heard by A?

3. Find the capacity in quarts of a pail in the form of a right cylinder 6.5 in. in diameter and 8.7 in. high.

4. Find the capacity in pecks of a cylindrical box $18\frac{3}{4}$ in. in diameter and $24\frac{1}{2}$ in. deep.

5. A stack of hay in the form of a cone whose diameter is 10 ft. must be how high to contain 100 cu. yd.?

6. If a cubic foot of a certain kind of stone weighs 160 lb., how much will a cubical block of the same kind of stone weigh whose edge is $2\frac{1}{2}$ ft.? How long is the edge of a cubical block of stone that weighs 600 lb.?

7. If a cubic inch of water weighs 252.722 grains, what must be the edge (inside measure) of a cubical vessel that will contain 80 pounds of water?

8. How many barrels of apples will it take to fill a bin 18 ft. long, 6 ft. wide, and $4\frac{1}{2}$ ft. high?

9. Gold melts at about 1200° C. What is that Fahrenheit?

10. Wrought iron melts at about 1600° C. What degree F.?

11. A stone dropped from a balloon reached the ground in 7 seconds. How high was the balloon?

12. The Liberal Arts building at the Chicago World's Fair was 210 ft. in height. How long would it take a body to fall to the floor? The Eiffel Tower at the Paris Fair was 985 ft. in height. How long would a body be in falling from the top?

13. What is the length of a pendulum that beats half seconds? that makes one vibration in two seconds?

14. *A pendulum* is 4.77 + in. long. How often will it vibrate?

APPENDIX.

WEIGHTS AND MEASURES.

(For Tables in Weights and Measures, see last pages of Book No. VII.)

The Gunter's chain consists of 100 links of 7.92 in. each. Surveyors generally use a steel ribbon, commonly 100 ft. in length, divided into feet and tenths and hundredths of feet.

A *hand*, used for measuring the height of horses at the shoulder, = 4 in. A *span* = 9 in. A *stone* of iron or lead = 14 lb. 1.15 common miles = 1 *geographical* or *nautical mile*, sometimes also called a *knot*. The length of a *degree of latitude* at the equator is 68.72 miles ; in middle latitudes from 68.9 to 69.05 miles ; in Polar regions about 69.3 miles. The length of a *degree of longitude* on the equator is 69.16 miles; on the 20th parallel about 65 miles; on the 40th parallel about 53 miles; on the 60th parallel about 34 miles; and on the 80th parallel about 12 miles.

An *ohm* is the standard measure of electrical resistance.

A square measuring 208.71 ft. on each side contains an acre. A *square* for measuring floors, etc., = 100 sq. ft.

The *long ton* (2240 lb.) is used in weighing iron and coal at the mines and goods at the custom houses. 1 lb. Avoirdupois = 7000 grains; 1 lb. Troy = 5760 grains.

The *carat* used in weighing diamonds, etc., = 3⅛ gr. Troy. The word carat is also used to express the proportionate fineness of gold (24ths) in jewelry, etc. The standard purity of gold and silver coins is .9 pure metal and .1 alloy, the alloy being pure copper.

1 lb. Avoirdupois = 27.7 cu. in. of distilled water at maximum density. 1 bu. grain, etc., = 2150.42 cu. in. 1 bu. vegetables, etc. (heaped), = 2688 cu. in. 1 gal. = 231 cu. in. In estimating contents of cisterns, etc., 1 barrel = 31½ gal., and 1 hogshead = 2 barrels.

1 *solar year* = 365 da. 5 h. 48 min. 49.7 sec. All years divisible by 4, except centennial years not divisible by 400, are leap years.

A book made by folding sheets 26 in. by 23 in. into 2 leaves is a *folio;* into 4 leaves is a *quarto;* into 8 leaves is an *octavo;* into 12 leaves is a *duodecimo*. Size of writing paper : billet note, 6 in. × 8 in.; commercial note, 8 in. × 10 in.; letter, 10 in. × 16 in.; foolscap, 12½ in. × 16 in.

1 *firkin* of butter = 56 lb. 1 bbl. of flour = 196 lb. 1 bbl. of beef or pork = 200 lb. 1 bbl. of salt = 280 lb. 1 *cental* of grain or *quintal* of fish = 100 lb. 1 cask of lime = 240 lb. 1 keg of nails = 100 lb.

A common teaspoon holds about 45 drops or 1 fluid dram. A common teacup holds about 4 fluid ounces. A common tablespoon holds about half a fluid ounce. A pint of water weighs nearly a pound.

A horse-power in machinery is estimated at 33,000 lb. raised 1 foot every minute.

A roof is said to be quarter pitch when the height of roof is one-fourth the width of *the building.*

Measurement of Lumber.

A *foot of lumber*, called also *board foot*, is 1 ft. long, 1 ft. wide, and 1 in. thick. Lumber less than 1 in. thick is considered an inch thick in measuring. Fractions of an inch in the width to be omitted, and the nearest integer taken. A *bundle of shingles* contains 250 shingles of an average width of 4 in. The bundle is 20 in. wide and has 25 courses on each side. Allowing for waste, 4 bundles will lay 100 sq. ft. laid 4 in. to the weather. 800 shingles laid 4½ in. to the weather will cover a square. A *bundle of laths* consists of 100 laths, each 4 ft. long, 1½ in. wide. Allowing for waste, 1 bundle of laths laid ¼ in. apart will cover 7 sq. yd.

Measurement for Plastering, Papering, etc.

Plastering, painting, and kalsomining are usually computed by the square yard, payment being generally allowed for half of the openings. American wall-paper is 18 in. wide and has 8 yd. in a single roll and 16 yd. in a double roll. Borders or friezes are sold by the yard.

Measurement for Stone and Brickwork.

Custom varies as to allowance for corners and for openings in estimating material and labor for stonework. Generally in estimating labor the corners are measured twice, and one-half of the openings is deducted. Brickwork is estimated by the thousand, 22 common bricks to the cubic foot. Bricks vary in size, the common bricks being 8 in. by 4 in. by 2 in.

Practical Measurements.

To find the :

Area of a parallelogram: Multiply the base by the altitude.

Area of a triangle: Multiply the base by half the altitude.

Area of a trapezoid: Multiply half the sum of parallel sides by the altitude.

Area of any polygon: Divide into triangles and find the sum of their areas.

Area of a circle: Multiply the square of radius by 3.1416.

Circumference of a circle: Multiply the diameter by 3.1416.

Diameter of a circle: Divide the circumference by 3.1416.

Hypothenuse of a right-angled triangle: Extract the square root of the sum of the squares of the base and perpendicular.

Base or perpendicular of a right-angled triangle: From the square of the hypothenuse subtract the square of the given side and extract the square root of the remainder.

Volume of a prism or cylinder: Multiply the area of the base by the altitude.

Convex surface of a right prism or cylinder: Multiply the perimeter of base by height.

Volume of a pyramid or cone: Multiply the area of the base by one-third the altitude.

Convex surface of a pyramid or cone: Multiply the perimeter of the base by half the slant height.

Volume of the frustum of a regular pyramid or cone: Multiply the sum of the areas of the two bases plus the square root of their product by one-third the altitude.

Convex surface of the frustum of a regular pyramid or cone: Multiply half the sum of the perimeters of the two bases by the altitude.

Surface of a sphere: Multiply the area of a great circle of the sphere by 4.

Volume of a sphere: Multiply the convex surface by one-third of the radius.

Mean diameter of a cask or barrel: Add to the head diameter two-thirds of the *difference* between the head and bung diameters.

Approximate Measurements.

A cord of stone will make about 100 cu. ft. of wall. 3 bu. of lime and 1 cu. yd. of sand will lay a cord of stone.

A bushel stricken measure contains about $1\frac{1}{4}$ cu. ft. The bulk of the bushel heaped measure is $\frac{1}{4}$ greater than that of the stricken measure.

A gallon contains about $\frac{1}{8}$ of a cubic foot. Six quarts dry measure are equal to nearly 7 quarts liquid measure. A barrel of $31\frac{1}{2}$ gal. contains about $4\frac{1}{2}$ cu. ft. A ton of hay occupies from 450 to 550 cu. ft. A ton of coal occupies from 34 to 36 cu. ft.

Weight of a Bushel in Pounds Avoirdupois.

In the second column the weight as fixed by law in a majority of States is given. In the third column exceptions are noted.

Commodities.	Pounds per Bushel.	Exceptions.
Apples, etc.	48	Me., 44; Vt., 46; N. J., 50; Or., 45; Wis., 57.
Barley	48	Cal., 50; Ga., Ky., Md., and Pa., 47; Or., 46; Wash., 45.
Beets	60	Mont., Wash., and Wis., 50.
Bran	20	
Carrots	50	Conn., 55.
Coal	80	Ky. (anthracite), 76; O. (bituminous), 70.
Corn (shelled)	56	Ar., 54; Cal., 52; N. Y., 58.
Corn (on cob)	70	Ind. 68.
Corn (meal)	50	Del., Ga., Ill., and D. C., 48.
Oats	32	Me., N. H., N. J., and Pa., 30; Md., 26; Or. and Wash., 36.
Potatoes	60	O., 58; Wash., 50 (sweet) generally less.
Rye	56	Cal., 54; La., 31.
Salt	50	Mass., 70; Mich., 56; Pa. (fine), 62.
Wheat	60	

A Table of Specific Gravities.

Air	.001292	Ice	.93
Alcohol (pure)	.80	Iron (cast, average)	7.21
Butter	.94	Lead	11.35
Bone	1.66	Limestone (average)	2.70
Brass (average)	8.38	Lime	.804
Brick (average)	2.09	Milk	1.03
Clay	2.16	Salt	2.13
Coal (anthracite, average)	1.56	Sand (average)	1.65
" (bituminous, average)	1.18	Sea-water	1.02
Copper (average)	8.75	Silver	10.5
Cork	.24	Steel	7.81
Glass (average)	2.76	Wood — Pine (average)	.49
Gold	19.30	Oak	.85
Human body (alive)	.89	Lignum vitæ	1.34

Value of Foreign Coins in United States Money.

Pound sterling	$4.866½	Dutch florin	$0.402
German mark	.238	Indian rupee	.252
French franc, Italian lira	.193	Russian gold rouble	.772
		Austrian crown	.203

Chinese tael . . . . . $0.784.

Table showing Rate of Interest allowed in the Various States.

(From the "World Almanac" for 1894.)

The legal rate is given in the first column. The rate allowed by contract is in the second column. * = any rate.

States.	Rate per cent		States.	Rate per cent		States.	Rate per cent	
Alabama	8	8	Louisiana	5	8	Ohio	6	8
Arizona	7	*	Maine	6	*	Oklahoma	7	12
Arkansas	6	10	Maryland	6	6	Oregon	8	10
California	7	*	Massachusetts	6	*	Pennsylvania	6	6
Colorado	8	*	Michigan	6	8	Rhode Island	6	*
Connecticut	6		Minnesota	7	10	South Carolina	7	8
Delaware	6	6	Mississippi	6	10	South Dakota	7	12
Dist. of Columbia	6	10	Missouri	6	8	Tennessee	6	6
Florida	8	10	Montana	10	*	Texas	6	10
Georgia	7	8	Nebraska	7	10	Utah	8	*
Idaho	10	18	Nevada	7	*	Vermont	6	6
Illinois	5	7	New Hampshire	6	6	Virginia	6	6
Indiana	6	8	New Jersey	6	6	Washington	8	*
Iowa	6	8	New Mexico	6	12	West Virginia	6	6
Kansas	6	10	New York	6	6	Wisconsin	7	10
Kentucky	6	6	North Carolina	6	8	Wyoming	12	*
			North Dakota	7	12			

Table showing the Amount of $1 at Compound Interest.

Years	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060000	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196	2.252192
13	1.468534	1.563956	1.665073	1.772196	1.885649	2.132928	2.409845
14	1.512590	1.618694	1.731676	1.851945	1.979932	2.260904	2 578534
15	1.557967	1.675349	1.800943	1.935282	2.078928	2.396558	2.759031
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.652848	1.794675	1.947900	2.113377	2.292018	2.692773	3.158815
18	1.702433	1.857489	2 025816	2.208479	2.406619	2.854339	3.379931
19	1.753506	1.922501	2.106849	2.307860	2.526950	3.025599	3.616526
20	1.806111	1.989789	2.191123	2.411714	2.653298	3.207135	3.869684

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